THE PROCEEDINGS OF
THE PHYSICAL SOCIETY

VOL. 49, PART 2 *March* 1, 1937 NO. 271

A CIRCUIT FOR COUNTING IMPULSES AT HIGH SPEEDS OF COUNTING

BY J. H. E. GRIFFITHS,
The Clarendon Laboratory, Oxford

Communicated by Prof. F. A. Lindemann, 1 *July* 1936. *Read* 11 *December* 1936

ABSTRACT. A simple circuit is described which gives one impulse to a recording meter for, say, 10 pulses from a Geiger-Müller counter or similar instrument. The limitations and conditions of operation are discussed, and it is shown that a correction of 10 per cent has to be applied at a counting speed of 3000 per minute.

§ 1. INTRODUCTION

It is often of great advantage to be able to use a high rate of counting when recording the impulses from a Geiger-Müller counter or similar instrument. The scale-of-two counter designed by Wynn-Williams(1) is excellent for this purpose, but it is rather costly to build.

This paper describes a simple method of integrating the effect of, say, 10 pulses to give one count on the recording meter. Although the resolving time is about 10 times that of the scale-of-two counter, it is sufficient for most work with Geiger-Müller counters, since a counting speed of up to 3000 per minute can be used, and it has the advantage of great simplicity and low cost. The error, which in principle might be as great as 10 because digits are not recorded, is negligible compared with the statistical error for reasonably large numbers, and can be avoided by measuring the time needed to count a fixed number instead of the number of impulses in a fixed time.

§ 2. CIRCUIT

The circuit is shown in figure 1. At each pulse delivered by the amplifier the thyratron discharges the condenser C_1 which is then recharged through R. The voltage V_2 across the large condenser C_2 rises until it becomes sufficient to discharge the neon tube T. V_2 then drops to the extinguishing potential of the neon tube and if this change of potential is v, the quantity of charge is equal to $C_2 v$.

Let V_1 be the difference between the original potential across C_1 and the final potential at which the thyratron extinguishes. Then $C_1 V_1$ is the quantity passed through the thyratron at each pulse and n pulses will discharge the neon tube once if

$$nC_1V_1 = C_2 v.$$

This may more properly be written

$$nC_1V_1 > C_2v > (n-1)\,C_1V_1 \qquad \text{......(1)},$$

an equation* which shows that considerable latitude is allowed in the operation of the circuit.

It is very important that the insulation of the condensers C_1, C_2 and C_3 should be as good as possible; for if there is a leak from the anode circuit of the thyratron to earth, charge will be lost from C_2. This will be equivalent to one pulse per minute if $C_1V_1 = 60\,V_1/R'$, where R' is the resistance to earth. This gives R' about $= 10^{10}\omega$. This order of insulation can be obtained with commercial condensers; the smaller condensers C_1 and C_3 should have mica insulation.

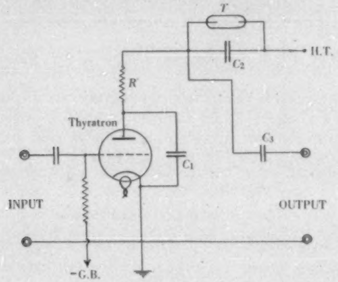


Figure 1.

However, it is only when the loss of charge during the time between two discharges of the neon tube is equivalent to one pulse that any error is caused. Thus with the resistance given above if 10 pulses occur in 30 sec., which is a very low rate of counting, the loss of charge due to leak is equivalent to half a pulse and is not recorded and, moreover, is not carried on to the next 10 pulses. Thus at higher speeds of counting a considerably higher leak can be allowed without any error being caused, and at low speeds single counting should be used.

Since the thyratron T is biased about 40 V., negative, the current to the anode is negligible.

3. THE RESOLVING TIME

There are two factors which give the resolving time, or rather the correction to be applied for pulses missed, and determine the value of R.

(1) During the time t_1 in which the grid of the thyratron is positive only a negligible amount of charge should pass through the resistance R. "Negligible"

---

* V_1 varies by the potential-difference between the sparking and extinguishing potentials of the neon tube, and this equation should be written

$$\sum_1^n C_1V_1 > C_2v > \sum_1^{n-1} C_1V_1.$$

The difference is small and the simpler equation is sufficiently accurate.

here may be taken to mean "less than V_1C_1/n". Assuming C_1 to discharge instantaneously the amount of charge is

$$t_1\frac{V_1}{R}.$$

Hence
$$t_1\frac{V_1}{R}<\frac{V_1C_1}{n},$$

or
$$nt_1<RC_1 \qquad \text{......(2).}$$

This factor is only of importance if t_1 is not the same for every pulse.

(2) The time t between two pulses must be sufficient to allow the condenser C_1 to be completely recharged. If there are N_0 pulses per second the average time τ between pulses is $1/N_0$ and the probability that after the arrival of one pulse the next pulse will arrive in the interval between t and $t+dt$ is

$$e^{-t/\tau}.dt.$$

If V_3 is the extinguishing potential of the thyratron, the charge in C_1 after a time t is

$$V_3C_1+V_1C_1(1-e^{-t/RC_1}),$$

and the deficiency of charge, i.e. the amount by which the charge on C_1 differs from the maximum value $(V_1+V_3)\,C_1$ is

$$V_1C_1.e^{-t/RC_1}.$$

The average deficiency of charge is given by

$$\frac{V_1C_1\int_0^\infty e^{-t/RC_1}.e^{-t/\tau}dt}{\int_0^\infty e^{-t/\tau}dt}=V_1C_1\frac{1}{\tau/RC_1+1}.$$

The operating conditions should be such that C_2v is only just greater than $(n-1)C_1V_1$ (see equation (1)) for in that case an amount of charge almost equal to C_1V_1 can be lost through partially unresolved pulses without any error occurring. The correction will therefore be 10 per cent when

$$nV_1C_1.\frac{1}{\tau/RC_1+1}=V_1C_1$$

or
$$\tau=(n-1)\,RC_1>n\,(n-1)\,t_1.$$

and will decrease rapidly at lower counting speeds owing to the smoothing effect of taking ten impulses at a time. Thus the effective resolving time is considerably longer than the duration of the pulse and is determined by it.

For this reason an amplifier giving equal pulses of short duration, such as that designed by Barasch(2), should be used. The Barasch amplifier gives a pulse-duration of about 10^{-4} sec., which allows a resistance of $2\cdot10^5\,\Omega$. to be used if $C_1=0\cdot008\,\mu$F. τ is then $1\cdot4.10^{-2}$ sec. for a 10 per cent correction, and N_0 about 50 per second or 3000 per minute. This compares with a correction of 10 per cent at 300 per minute when using a mechanical counter with a maximum speed of 1500 per minute.

The complete unit for recording either singly or in multiples of 10 is shown in figure 2.

The switch S connected the input with either V_1 for multiple counting, or V_2 for single counting. The neon tube used is a Phillips neon indicator of the type with leads at opposite ends. The caps and resistance were removed.

To adjust the circuit for a given value of n (say 10) a change of C_1 gives a coarse adjustment, and the potentiometer P a fine adjustment. The potentiometer controls the potential across C_1, since the potential across C_2 is only controlled by the neon tube. Adjustment is best carried out by counting aurally the number of pulses for each discharge of the neon tube at a low rate of counting. The potentiometer is then set so that this number is 10, while for a slight increase of voltage it is 9.

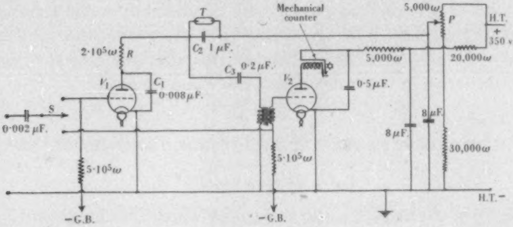


Figure 2.

Two 8-μF. electrolytic condensers are used to prevent the large pulse given by the thyratron T_2 from disturbing the first circuit. The circuit could probably be improved by using in place of R a valve or photocell or some device to ensure that current only flows into C_1 after the thyratron has extinguished itself. A larger multiplication than 10 might then be used with profit.

§ 4. ACKNOWLEDGEMENT

In conclusion I should like to thank Prof. Lindemann for his helpful advice and encouragement.

REFERENCES

(1) Wynn-Williams, C. E. *Proc. roy. Soc.* A, **136**, 312 (1932).
(2) Barasch, H. P. *Proc. phys. Soc.* **47**, 824 (1935).

THE ELECTRICAL CONDUCTIVITY OF THIN FILMS OF THE ALKALI METALS SPONTANEOUSLY DEPOSITED ON GLASS SURFACES

By A. C. B. LOVELL,

H. H. Wills Physical Laboratory, University of Bristol

Communicated by Prof. A. M. Tyndall, 5 November 1936. *Read in title* 22 *January* 1937

ABSTRACT. A description is given of an investigation of the electrical conductivity of the invisible thin films of the alkali metals which are deposited spontaneously on the walls of an enclosure in which they are sealed *in vacuo*. The resistivity of the films decreases in the order potassium, rubidium, caesium. The resistivity of the rubidium film is in reasonable agreement with a previous measurement given by Ives and Johnsrud. Some evidence is adduced in support of the view reached by the above workers that the thicknesses of these spontaneous deposits are of the order of those of monatomic layers.

---

§ 1. INTRODUCTION

This paper describes a detailed investigation of the electrical conductivity of the invisible thin films of the alkali metals which are deposited spontaneously on the walls of a vessel containing the metal *in vacuo*. The term *spontaneous deposition* will be confined to the case when the surface is either at the same temperature as the vapour to which it is exposed or at a somewhat higher temperature. A full account of another investigation by the writer on the conductivity of films of known thickness deposited under controlled conditions on cooled surfaces is in course of publication elsewhere(6, 7).

The first description of this phenomenon of spontaneous deposition occurs in the work of Elster and Geitel(1, 2), who observed that in photocells containing the alkali metals the anode showed photoelectric activity although previously cleaned of any deposit by heating.

The photoelectric properties of the films spontaneously deposited on metal surfaces were investigated by Ives(3), and Ives and Johnsrud(4) studied films of rubidium deposited on glass surfaces. It was observed that the films of rubidium on glass were electrically conducting and in darkness gave a pure ohmic resistance of $1{\cdot}3 \,.\, 10^{8}$ ohms on a plate measuring $1{\cdot}9 \times 4{\cdot}0$ cm. This resistance decreased under illumination as a result of the liberation of photoelectrons forming an added current along the tube under the potential gradient (the photoresistance effect).

Ives and Johnsrud(5) attempted to determine the thickness of these spontaneous deposits by measuring the shift in the phase and azimuth of polarized light reflected from the surface as a result of the presence of the film. This indicated a thickness of the order of 6 A., although the authors point out that the accuracy of the experiment was not sufficient to determine the thickness within a factor of 10. A thickness of this order would give a resistivity of the order of 10 ohm-cm., as compared with $11{\cdot}3 \times 10^{-6}$ ohm-cm. for rubidium in bulk at 0° C.

§ 2. THE EXPERIMENTAL TUBE

The spontaneous deposition occurs on the tube *A* (of soda or pyrex glass), which fits closely over the pyrex inlet *B* from the main body of the tube, figure 1. The temperature of the surface *A* can then conveniently be controlled by insertion of a heater into *B*. The deposition normally occurs over all the tube walls, and the resistance measured would be that of the tube *A* together with the parallel resistance across the outgoing leads. This is prevented by placing heaters over the outlet pinches *CC* and using an earthed guard ring *G*. The presence of thermocouples on the surface *A* gives rise to similar difficulties, and for this reason the temperature of *A* was calibrated against heater current in dummy experiments and the thermocouples were then removed.

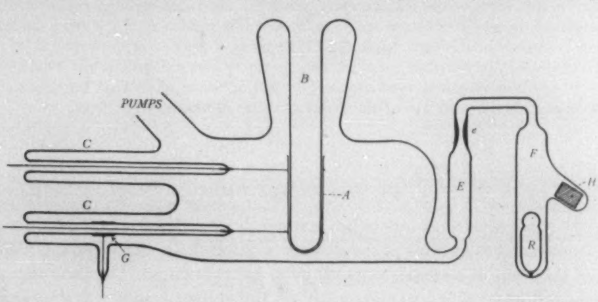


Figure 1.

Contacts with the film were made by hoops of platinum wire partially embedded in the tube *A* for soda glass; and for pyrex glass by baking on small rings of colloidal graphite and wrapping platinum wire around. The whole tube was immersed in a thermostatically controlled oil bath; in this way the temperature of the alkali metal on the tube walls was maintained constant to within $\frac{1}{10}$° C. whatever the temperature of the surface *A* which, as was stated above, is independently controllable.

§ 3. INITIAL OBSERVATIONS

The initial experiments were concerned with attempts to reproduce the observations of Ives and Johnsrud(4) for rubidium on glass. The rubidium was prepared by reducing the chloride with calcium in a side tube, distilled into the experimental tube and the reaction bulb sealed off. With this arrangement it proved difficult to reproduce the results obtained by Ives and Johnsrud; on one or two occasions the films formed capriciously, but for a considerable time the causes of failure remained obscure. Eventually the failure to obtain conducting films was traced to an effect of the gas evolved during the reaction of the calcium with the rubidium chloride. It appears that foreign gas layers were put down on the surface at *A* and these inhibited the formation of conducting films. When the rubidium was separately prepared and thoroughly outgassed before being put into the experimental tube, the films were readily obtained.

§ 4. PREPARATION OF THE ALKALI METALS

Metallic rubidium and caesium have been prepared by the reduction of the respective chlorides with calcium metal *in vacuo*, the reaction being carried out in a separate vacuum apparatus. After the apparatus had been thoroughly baked out the metals were distilled successively through a train of bulbs into a group of glass ampoules, which could be sealed off at a constriction and stored for use. In the case of potassium the metal itself was distilled in a similar manner. When the alkali was to be introduced into the experimental tube one of these ampoules *R* was inserted into the tube *F*, figure 1, and broken by means of the magnetic slug *H*, and the metal was distilled into *E*. A constriction *e* then enabled the tube *F* to be sealed off. In this way the dangers referred to in § 3 were avoided. The alkali was then distilled over to cover the walls of the tube.

Prior to the breaking of the ampoule the whole tube was baked out at 500° C. for about 6 hours, the pumping tubes being thoroughly torched at the same time. The tube was continuously evacuated by a mercury diffusion pump working through a liquid-air trap. The first stages of the bake-out were carried out without liquid air over this trap in order to avoid condensation of excessive amounts of impurity. Once the bake-out had been accomplished the very greatest care had to be exercised to prevent the entry of mercury vapour into the tube.

§ 5. MEASUREMENT OF RESISTANCE

The resistance of the tube *A* was measured by means of a potentiometer and galvanometer. The current through the unwanted parallel resistance forming around the tube walls was short-circuited to earth by the guard ring *G* used in a standard manner. With 100 V. across the film it was possible to detect a resistance of 10^{13} ohms.

§ 6. RESULTS

In the initial distillation of the alkali from *E* into the experimental tube, condensation naturally occurs over the tube walls and over the surface of *A*, giving a resistance corresponding to that of the bulk metal. The alkali is cleared from *A* by placing the heater in *B* at about 150° C., and the resistance of *A* is then infinite. When *A* returns to the temperature of the oil bath the spontaneous film begins to form. A curve exhibiting the relation between resistance and time* can thus be plotted. Such a curve for caesium with the oil bath at 20° C. is shown in figure 2. It will be seen that the final resistance approaches a steady value. The formation of the film is complete in about 4 hours for caesium, and in 10 hours for rubidium. The curves of formation can be repeated by clearing the film from *A* by the heater in *B*. If, however, when the alkali is cleared from *A* the temperature of this surface is then lowered but adjusted to a value somewhat higher than that of the oil bath, then the spontaneous film comes to equilibrium more quickly but the final resistance is higher. The complete curves showing the logarithm of the resistance against temperature for a given vapour pressure of the alkali are shown in figure 3 for caesium and rubidium.

---

* The dimensions of the tube on which these measurements were made were as follows: diameter 16·8 mm., length 29·0 mm., giving the resistivity ρ as equal to $R \times 1{\cdot}82 \times$ the thickness of the film.

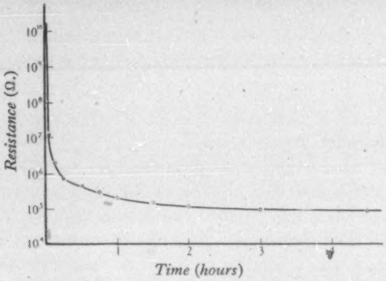


Figure 2. Formation of spontaneously deposited caesium film on soda glass.

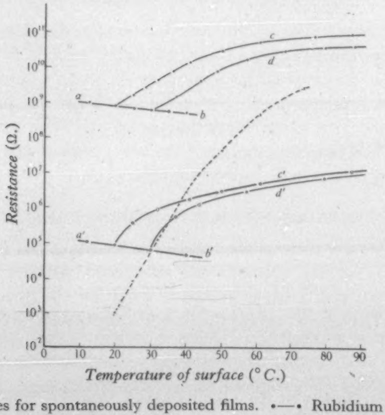


Figure 3. Curves for spontaneously deposited films. •—• Rubidium on soda glass; ×—× caesium on pyrex glass; ⊙—⊙ caesium on soda glass.

The lines *ab*, *a′b′* represent the uniform-temperature equilibrium points for these films, since they occur when the whole tube is at a uniform temperature, i.e. when the surface on which condensation occurs is at the temperature of the surroundings. It must be emphasized that the temperature conditions involved in these equilibrium points are critical, since if the temperature of the surface A falls slightly below that of its surroundings then normal condensation of the alkali occurs, giving a very low resistance.

The remaining curves c, c', d, d' represent deposition on a surface at a higher temperature than its surroundings. (The oil bath keeps the alkali on the tube walls at a constant temperature, independent of the temperature of A, thus giving a constant vapour pressure.) Two series have been taken, c and c' with the oil bath at 20° C., and d and d' with it at 30° C.

It has been mentioned above that equilibrium is reached much more quickly when the surface A is at a higher temperature. For example, with caesium at 20° C. in the tube the formation of the film is completed in ¾ hour when the surface A is at 90° C. In the case of rubidium the time is 2 hours, and these times may be compared with 4 and 10 hours respectively for the uniform-temperature equilibrium points. Throughout the range of temperatures investigated, namely 20–90° C., the time for attaining equilibrium diminished steadily as the temperature of A increased, though the equilibrium resistance rose steadily of course, as is shown in figure 3. In view of its high uniform-temperature equilibrium resistance (5×10^9 ohms) potassium has not been fully investigated.

The above instances relate to deposition on soda glass surfaces. In the case of pyrex surfaces the curves are much steeper. An example for caesium with the vapour pressure at 20° C. is shown by a broken line in figure 3. In all cases the measurements have been carried out in darkness to obviate the photoresistance effect.

§7. DISCUSSION

These spontaneously deposited films of the alkalis are quite invisible, and the only evidence as to their thickness is that found by Ives and Johnsrud(5). From the following considerations the present investigation gives some evidence in support of their view that the films are monatomic layers.

The uniform-temperature equilibrium resistance for rubidium on soda glass at 20° C. was found in § 6 to be 8×10^8 ohms, on the tube whose measurements are given in that section. Taking Ives and Johnsrud's estimate of the thickness as 6 A. this gives the resistivity ρ as 90 ohm-cm.* For caesium on soda the resistivity is about 10^{-2} ohm-cm. and on pyrex 10^{-4} ohm-cm., the same estimate of the thickness being again used.

Now an investigation has been made by the writer of the resistivity of alkali metal films of known thickness deposited on a cooled surface under controlled conditions(6, 7). Stable or unstable films could be obtained according to the conditions, instability being characterized by a higher resistivity which increased with the time during which the films were kept *in vacuo* after deposition. Although the conditions obtaining in the course of that work were quite different from those described in the present paper, it may be pointed out that for films deposited under the worst conditions of instability resistivities of the order given above were only obtained in rubidium films having a thickness of less than 10 A. The resistivities for caesium similarly correspond to those of the films of about 3 and 10 A. respectively.

---

* In view of the uncertainties involved this shows good agreement with the resistivity of the order of 10 ohm-cm. given by Ives and Johnsrud(4) for these films, if similar assumptions are made as to their thickness.

Thus the evidence drawn from a comparison of resistivities tends to indicate that the thickness given by Ives and Johnsrud is certainly not in error by as much as a factor of 10; it thus seems reasonable to suppose that these spontaneous deposits are approximately monatomic layers.

Although there is as yet no evidence to enable speculations to be made as to the nature and structure of these spontaneously deposited films, it is of interest to point out that the present work has fitted in with the hypothesis developed in reference(7), concerning the relative instability of a given film of the three alkalis. It was found that as one passed down the series potassium, rubidium, caesium the stability of a film of given thickness steadily increased, and it was suggested that this might be accounted for by the atoms of alkali adhering more and more closely to the glass in this order. In the present work on spontaneous deposits it has been seen that the resistivities of the three alkalis decrease in the order potassium, rubidium, caesium, which would correspond to increasing coherency of the films.

§ 8. ACKNOWLEDGEMENTS

The author wishes to express his indebtedness to Dr E. T. S. Appleyard for suggesting the problem and for his continued interest. Also to the Department for Scientific and Industrial Research for a maintenance grant.

REFERENCES

(1) Elster and Geitel. *Phys. Z.* **21**, 361 (1920).
(2) Elster and Geitel. *Ann. Phys.*, Lpz., **67**, 420 (1922).
(3) Ives. *Astr. J.* **60**, 209 (1924).
(4) Ives and Johnsrud. *Astr. J.* **62**, 309 (1925).
(5) Ives and Johnsrud. *J. opt. Soc. Amer.* **15**, 374 (1927).
(6) Lovell. *Proc. roy. Soc.* A, **157**, 311 (1936).
(7) Appleyard and Lovell. *Proc. roy. Soc.* (in the Press).

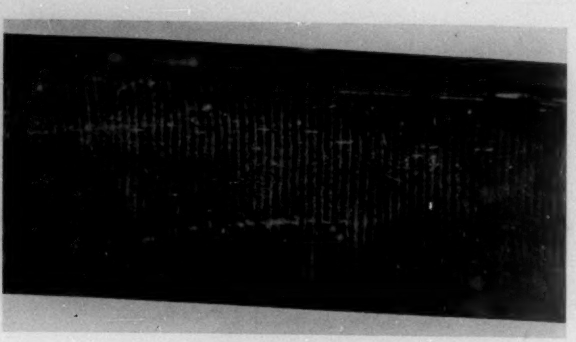


Figure 1. Magnification, × 2

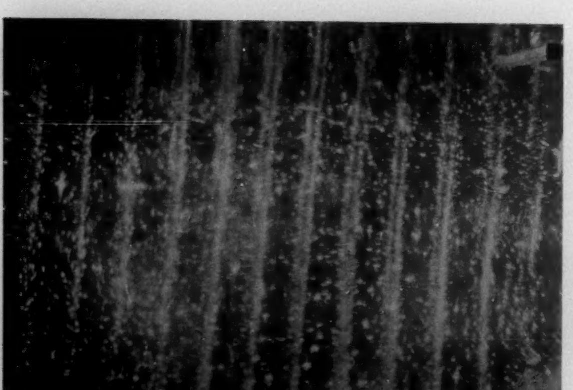


Figure 2. Magnification, × 100

THE SMOKE METHOD OF MEASURING SUPERSONIC VELOCITIES

By R. C. PARKER, B.Sc., Ph.D., University College, London

Received 31 *October* 1936. *Read in title* 27 *November* 1936

ABSTRACT. In view of certain criticisms brought against it, the smoke method used by Pearson has been further investigated. Various possible sources of error have been eliminated, and measurements of the velocity of sound have been made in air, oxygen and nitrogen in the frequency range 92·2 to 801·7 kc./sec. The results are believed to be correct to within 1 part in 3000, and no evidence of dispersion in this range has been found. An explanation is given for Pearson's results.

---

§ 1. INTRODUCTION

A new method for determining the velocity of sound at supersonic frequencies has recently been developed by Pearson(1). Results were obtained for frequencies in a range from 92·2 to 801·7 kc./sec. and showed definite evidence of dispersion. Certain criticisms have been brought against this method, and the purpose of these investigations has therefore been to repeat certain of the measurements under more precise conditions, with special attention to the points that have been called into doubt. The method has been extended and measurements on the velocity have been made in oxygen and nitrogen.

§ 2. PRINCIPLES OF THE METHOD

The method is essentially that of the familiar Kundt's tube. The length of an enclosed air column, energized by a piezo-electric quartz crystal oscillator, is adjusted for resonance conditions, and the position of the nodes in the standing wave system are marked by rings of coagulated smoke particles. The frequency n at which the quartz crystal vibrates is accurately known, and this with a measured value of the wave-length λ_t at a temperature $t°$ C. ($T°$ K.) enables the velocity V_t to be deduced. The velocity V_0 at 0° C. is obtained from the relation

$$V_0 = V_t \sqrt{\frac{T_0}{T}},$$

where, of course,

$$V_t = n\lambda_t.$$

The importance of this method ultimately depends upon the accuracy with which λ_t can be measured. By the introduction of certain modifications the smoke figures have been eventually obtained in a particularly clear manner. Figures 1 and 2 show rings formed at a frequency of 219 kc./sec. Their double structure is clearly visible and so confirms the theory that, at these frequencies, the particles act as obstacles to produce phenomena similar to those observed by Andrade(2) using cork dust at audio frequencies.

§ 3. MEASUREMENT OF λ

The values of V_0 given by Pearson, for each particular frequency, represents the mean of eight determinations. At a frequency of 310 kc./sec. the individual measurements of the velocity differed by 1·4 m./sec.; at other frequencies this difference was presumably greater. Such a scatter not only necessitates a large number of readings, but it suggests that there may be additional factors for which allowance has not been made. Pearson does not say what caused this scatter, though from the data supplied one can only attribute it to some error in λ_t. A preliminary set of readings made with the original apparatus confirmed this view. From eight determinations made at a frequency of 220 kc./sec. a maximum difference of 1·3 m./sec. was obtained. The wave-length was obtained by noting the position of forty nodal rings and treating the observations in the usual way, i.e. the readings were divided into two groups to give a series of values for $10\lambda_t$. The rings were so distorted and ill-formed that difficulty was experienced in setting the cross wire of the microscope accurately at their centre. The neighbouring values of λ_t often differed by 10 per cent, while the final mean had a probable error of $\frac{1}{4}$ per cent. From these readings no systematic variation in the ring-spacing was detected.

Rings of clearer definition were eventually obtained by establishing more stable conditions of oscillation, and by making sure that the plane of the crystal and piston-face were perpendicular to the axis of the tube. By means of a travelling microscope, which enabled settings to be made to within 0·002 mm., 40 rings were measured near the crystal. The readings of the nodal positions were again divided into two equal groups to give 20 values for $10\lambda_t$. The maximum divergence shown by these readings was only 0·3 per cent and a repetition of the experiment at the same temperature gave values of V_0 agreeing to within 0·1 per cent, the measurements being made, each time, along the same section of the tube.

The uniformity of the tube. The conditions which exist at supersonic frequencies make it difficult to assess, theoretically, the influence which any non-uniformity of the tube may have upon the nodal positions. It being now possible to measure the wave-length to a high degree of accuracy, an experimental investigation was made by extending the measurements along the length of the tube (35 cm.). The wave-length was seen to vary according to the portion of the tube over which measurements were made. From a minimum, which occurred near the centre of the tube, the value rose as either end was approached. The extreme values differed by 1 per cent. An explanation of this irregularity was sought by examining the tube for lack of uniformity. It was removed from the apparatus and its internal diameter measured at various points along its length. The uniformity of the tube was found to be poor; not only did it flare out at either end, but the diameter of the open end was 3 per cent in excess of that at the closed end. The cross-section was not circular, and four typical values for diameters inclined at $\pi/4$ through one section were 1·936, 1·942, 1·969 and 1·964 cm.

This tube was replaced by a German glass tube, of diameter 2 cm., specified as being of great uniformity. Measurements showed a cross-section which was circular to 0·02 mm. and a diameter constant along the tube to 0·01 mm. Rings formed in this tube were much more uniform in spacing. Groups of 40 rings measured at different sections along the tube gave values of λ_t constant to 0·4 per cent, while four independent determinations of the velocity made at a frequency of 127·5 kc./sec. showed a maximum difference of only 0·4 m./sec. For these latter values, groups of 20 rings were measured at about $200\lambda_t$ apart, the complete number of half wave-lengths separating these groups being found by obtaining an approximate value of λ_t from the sets of 20 rings.

Effect of amplitude. It had been suggested (3) that as certain crystals used by Pearson had broken on excitation, their amplitude must have been abnormally large. The appearance of the crystals did not lend support to this suggestion but invariably indicated a weakness at the groove cut for the supporting screws. This point was of importance, since change of velocity with amplitude furnished a possible explanation of any velocity increase: if the amplitude associated with any crystal was sufficiently large, the wave-front would become distorted to give an increase in the wave-velocity. Such an increase would naturally depend upon the condensation, as the velocity of a large amplitude wave is given by (4)

$$V = V_0 (1 + S)^{(1+\gamma)/2}$$

where V_0 is the velocity of the small amplitude wave, S the condensation and γ the ratio of the specific heats.

With the 219-kc./sec. crystal two measurements of the velocity were made for two different values of the amplitude, a quantity assumed to be proportional to the excitation voltage on the crystal face. The first measurements were made under the conditions formerly used, while for the second set the excitation voltage was reduced to one fifth. Now if, in the latter case, S was still large enough to cause an increase in the wave-velocity of 1/10 per cent, say, then for the case in which S was five times as great the increase in the wave-velocity would be 1 per cent. Values of V_0 for the two sets of measurements came out to be 331·7 and 331·9 m./sec., which values are substantially in agreement. Throughout the subsequent measurements the lower excitation voltages were employed, and thus there was no likelihood of any error greater than 1 in 1000 being introduced as a result of large velocity-amplitudes.

Concentration of smoke particles. Two objections may be raised against the use of smoke particles obtained from burning tobacco. In the first place the particles consist of minute drops of liquid formed on dust nuclei and so, on sedimentation, might appreciably increase the moisture-content of the air. Secondly, the particle concentration is relatively high and so might influence the wave-propagation by a loading effect. Such possibilities were obviated by using, instead of tobacco smoke, magnesium-oxide smoke at a very low concentration.

A known weight of magnesium ribbon (80 mg.) was burnt in a tank, and the smoke-laden air, after passing through a long calcium-chloride tube, was introduced into the apparatus. The volume of the tank was 5.10^5 cm.^{3} and thus the ratio of the aggregate mass of smoke particles per cm.^{3} to the mass of 1 cm.^{3} of air was 2.10^{-4}. This represents an upper limit, because the contents of the tank were stirred for

15 minutes, when the concentration decreased as a result of sedimentation and diffusion to the walls. Data on coagulation, published by Whytlaw-Grey and Patterson⁽⁵⁾, allow this decrease to be calculated, and show that after the 15 minutes stirring the ratio mass of smoke particles to mass of air becomes 10^{-6}. A still further reduction of smoke concentration occurs when the air passes through the calcium chloride tube. It would seem safe to conclude that an added mass of the above magnitude could not influence the propagation of sound in the tube to a measurable extent.

It is interesting to obtain an estimate of the number of particles per cm.³ Taking for the density of the magnesium oxide smoke particles a value of 0·35 g./cm.³ and taking a measured value for their radii of $3 . 10^{-5}$ cm. (see a forthcoming paper by Andrade and Parker) the concentration works out at 10^5 particles per cm.³ At these low concentrations and at the lower frequencies the sound vibrations had to be continued for times up to two hours to obtain satisfactory rings. At the higher frequencies, however, well defined rings were formed much sooner, so that at a frequency of 801·7 kc./sec. the process was complete in about five minutes.

§ 4. VELOCITY OF SOUND IN GASES

The preliminary experiments already described were all made in air at two frequencies, namely 127·5 and 219 kc./sec. It remained to cover the rest of the frequency range. It was also decided to measure the velocity of sound in oxygen and nitrogen, for this would serve the additional purpose of verifying any evidence of dispersion. The apparatus was redesigned, since leaks are only negligible when air is the gas under consideration. The same method was used throughout, except that, in the case of nitrogen, magnesium-oxide smoke was not used. It was replaced by magnesium-nitride smoke, which was obtained by heating magnesium ribbon electrically in an atmosphere of pure nitrogen. The smoke proved satisfactory, and its adoption eliminated the question of the removal of oxygen which would have been necessary with magnesium-oxide smoke.

§ 5. APPARATUS

A cross-sectional view of the apparatus is shown in figure 3. The glass tube *A* was supported by short sleeves centrally disposed within the end faces of an outer tank *G*. The outer tank, which served as a water bath to eliminate convection, was 48 cm. long and had a square cross-section of 6 cm. The sides of the tank were covered with plate-glass windows to facilitate viewing of the glass tube. Through one end plate passed a brass tube *H* carrying a plunger which was made to fit the inside of the glass tube. The end plate *M* was detachable and a collar *P* enabled the tube and plunger to be rigidly locked in any desired position. The smoke-laden gas entered the apparatus through the tubes *L* and *H* and finally passed into the resonance space through two holes suitably drilled through a piston, which in turn was contained in the brass plunger. By means of an end cap *K* the piston could be withdrawn into the plunger to present a plane reflecting face to the on-coming sound waves.

The protruding end of the glass tube was enclosed in a rectangular brass chamber of dimensions 6 × 6 × 8 cm. Its two side faces were fitted with glass windows, and two brass stop cocks, not shown, enabled it to be separately supplied with gas.

The method of mounting the crystal is shown in figure 4. The brass ring *B*, 2·7 cm. in internal diameter and 4 cm. in external diameter, carried three pointed screws which gripped the crystal around its central groove. The two lower screws were secured into position by locking-nuts, and a third, *D*, enabled the clamping pressure to be adjusted. The holder *B* was fixed to a short brass tube *Q*; this fitted loosely over the glass tube so that the crystals were held centrally in its open end. Openings were made in both the glass and the brass tubes to give entry to the small electrode, while two further openings in *Q* enabled the electrode to be viewed. The crystals varied in length from 2 mm. to 24 mm., and hence the position of the crystal-holder had to be adjustable. The screw *D*, figure 3, passed through a slot, 1 cm. long, cut in the upper face of the chamber, and then through a hole drilled in a lower plate *R*. The second plate *R* was arranged to slide flush with the inner surface of the top face, and so, the interface surface having been smeared with vacuum grease, the screw *D* could be used to move the crystal holder without leak. The sliding plate was secured to *C* by four corner flange screws moving similarly along slots cut in the upper face.

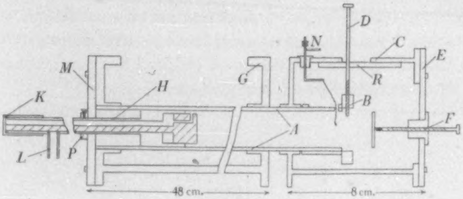


Figure 3.

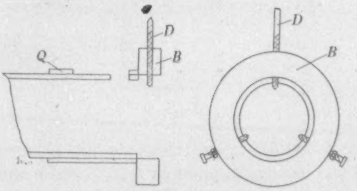


Figure 4.

The two electrode-supports entered by two ebonite bushes. The large electrode was a flat copper disc 2 cm. in diameter and was attached to the screw *F*, figure 3, by a small light copper spring. Its approach to the near coppered face of the crystal could be finely adjusted. For the smaller electrode a small piece of copper foil was soldered on to a light springy wire; this was attached to a rod *N*. The position of this small electrode was critical, and to obtain a comprehensive adjustment the rod was mounted to move vertically and to rotate.

With this modified apparatus it was found much easier to make the necessary adjustments that were required before the crystals would oscillate. In fact, difficulty was experienced with the crystal of highest frequency only; this was an important advantage, because formerly the crystals would occasionally stop oscillating, and in the process of getting them to start again the rings would become slightly distorted owing to uncertain circulations or through momentary oscillation at some neighbouring frequency. When the crystal which had a natural frequency of 801 kc./sec. was used, the effective tube length had to be reduced to 15 cm., otherwise no rings were formed on account of the high attenuation.

A diagrammatic sketch of the complete arrangement is shown in figure 5.

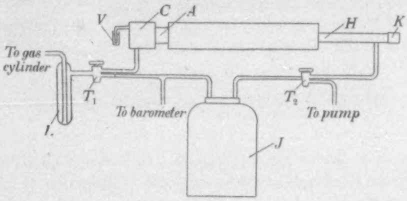


Figure 5.

The gas was admitted into the glass resonance tube *A* from a cylinder. Between the cylinder and *A* was inserted a liquid-air trap *L* and a stone jar *J*, closed by a metal cap cemented with sealing wax. The liquid-air trap served to remove any volatile gases present as impurities. In the stone jar was a length of magnesium ribbon which could be electrically heated to produce magnesium-oxide or magnesium-nitride smoke, according to which gas was used$^{(6)}$.

In carrying out a measurement, any residual gas from the previous experiment was first removed from *L* and *J* by means of a hyvac pump, and, when the pressure had fallen to a few millimetres of mercury, gas was admitted from the cylinder and allowed to stream through the glass resonance tube *A* for 30 minutes, after which time the air or gas initially present was judged to be completely replaced. The magnesium ribbon in *J* was then fired and the resulting smoke passed with the gas into the apparatus. When sufficient had entered, the gas stream was deviated by the tap T_1 and passed directly into the chamber *C*, where a slight excess pressure was maintained to prevent leaks round the electrode screws. A bubble indicator *V* gave a check on the rate at which the gas was flowing.

To measure the rings the subsidiary chamber *C* and the piston arrangement *H* were removed. Light from a 5-ampere arc was then passed axially down the tube, and the rings were thus viewed by scattered light against a dark background.

§ 6. RESULTS

The velocity of sound was determined in air, oxygen and nitrogen. These gases were contained in 100-ft.^{3} cylinders. The impurity in the nitrogen, supplied as oxygen-free, was 0·2 per cent and that in the oxygen 0·8 per cent. These impurities were due to the presence of the rare gases. The air supplied was atmospheric, carbon dioxide and water vapour being removed by the liquid-air trap and the drying-tube.

The velocity of sound was first determined in air. The value of V_0 was taken as the mean of four determinations. A representative example of such a set is given in table 1.

Table 1. (Frequency $n = 219{\cdot}28$ kc./sec.)

Temperature (° C.)	Wave-length (mm.)	V_0 (m./sec.)
17·2	1·5600	331·8
18·0	1·5611	331·6
15·2	1·5540	331·6
15·2	1·5551	331·9

Mean value of $V_0 = 331{\cdot}7$ m./sec.

The mean values of V_0 obtained at each of the other frequencies are given in table 2.

Table 2

n (kc./sec.)	Air	Oxygen	Nitrogen
	V_0 (m./sec.)	V_0 (m./sec.)	V_0 (m./sec.)
127·53	331·9	315·3	337·1
219·28	331·7	—	337·0
297·66	331·8	315·2	336·9
485·34	331·6	315·3	336·9
801·67	331·8	315·4	337·2

In table 2 are given the values of V_0 obtained in nitrogen and oxygen respectively. As there was no indication of any dispersion, the range of frequencies was, in the case of oxygen, covered by four crystals; while in view of the preceding agreement in the velocity-measurements only two determinations were made for each V_0.

In none of the gases did the velocity show any systematic variation, and hence final values of V_0, taken as the mean of the values obtained at all frequencies, are appended in table 3. The probable error in these mean values is 3 in 10,000.

Table 3. Mean values of V_0 (m./sec.)

Air	$331{\cdot}7_0$
Oxygen	$315{\cdot}3_0$
Nitrogen	$337{\cdot}0_2$

The Kirchhoff correction, which must be applied at sonic frequencies in order to get the value of the velocity in free space, was neglected. For the correction is never greater than 1 in 3000, and this, in view of the small trace of impurity in the gases, must represent the order of accuracy of the results.

The excess of V_0 above the normal value obtained by Pearson, particularly at the lower frequencies, would seem to be due to a confusion in the ring spacing mainly caused by some bad lack of uniformity of the tube near the piston. The writer made further measurements with the original tube and obtained values in good agreement with those of Pearson. Over this particular section of the tube the ring-formation was such that, in obtaining an estimate of λ_t, the rings were liable to be miscounted. A particularly clear, though distorted, set of rings was obtained and revealed a formation as shown in figure 6. Thus, for measurement made with the microscope along the bottom of the tube (i.e. within the region *AA*) a miscount would be made. An allowance for this miscount brought the values into line with those formerly given. For measurements made above a frequency 310·2 kc./sec. the tube-length was shortened, so that the above-mentioned section of the tube was not used; the values for V_0, with the original tube, were then found to be between 331·8 and 332·2 m./sec.

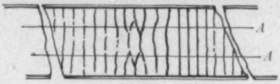


Figure 6.

§ 7. DISCUSSION OF RESULTS

Measurements on the velocity of sound at supersonic frequencies have, in recent years, been the subject of much interest. Since the first important measurements of the velocity in air by G. W. Pierce(7), the acoustical interferometer has been employed by many experimenters. The irregularities in the proximity of the source and the phenomena of multiple peaks and interferometer satellites have been studied in some detail, with the result that the method is now one of precision.

J. C. Hubbard(8) made a detailed examination of the method and concluded that for the frequencies he used, namely 218 to 476 kc./sec., there was no change in the velocity. W. H. Pielemeier(9), extending the frequency range to 1216 kc./sec., obtained values of V_0 lying between 331·6 and 331·7 m./sec., which values were also constant at all frequencies. Further evidence of independence of frequency was added by C. D. Reid(10), who worked between frequencies of 42 and 140 kc./sec.; his final value for V_0 was $331{\cdot}6_3$ m./sec. A slightly higher value of $331{\cdot}8_5$ m./sec. was obtained by Pan Tcheng Kao(11), whose frequency range extended from 20 to 70 kc./sec. W. D. Hershberger(12) made use of the interferometer to investigate the multiple-peak phenomena noticed by Pielemeier, Kao and others; he varied the amplitude of the exciting e.m.f. within wide limits and found several attributable causes for the previously observed phenomena. He made measurements on the velocity between 20 and 70 kc./sec. and showed V_0 to be independent of the frequency. Recent work by M. Grabau(13) and G. A. Norton(14), though similarly restricted to frequencies between 20 and 70 kc./sec., show values for V_0 in excellent agreement, their respective values being 331·68 and 331·76 m./sec.

In the region of sonic frequencies W. G. Shilling and J. R. Partington(15), using the resonance-tube method, obtained the lower value of 331·4 m./sec. The measurements were made at a frequency of 3 kc./sec. Their method of applying the Helmholtz-Kirchhoff correction, particularly at high temperatures, was criticized by R. E. Cornish and E. D. Eastman(16) who, employing a similar method, deduced a value of V_0 equal to 331·5 m./sec. C. B. Vance(17) used a resonance-tube method in which lycopodium powder was used to indicate the nodal positions; he made measurements between 30 and 200 kc./sec., and to investigate various tube corrections he chose for the free velocity in air a value of 331·77 m./sec. G. W. C. Kaye and G. G. Sherratt(18), used the resonance-tube method and made measurements from 500 c./sec. to 27 kc./sec., the velocity remained constant and equal to 331·6 m./sec. Unpublished experiments of E. N. da C. Andrade, D. H. Smith and S. H. Wheeler give a value of 331·74 m./sec. for frequencies between 800 and 1400 c./sec.

The only definite evidence of dispersion existing at normal temperatures seems to be afforded by C. Ishii(19) who used frequencies up to 3000 kc./sec. From 300 to 2000 kc./sec. a normal value of 331·6 m./sec. was obtained, but at 2892 kc./sec. it had risen to 332·6 m./sec. No other worker seems to have extended the range so far.

From the theoretical work of Kneser(20), no dispersion would be anticipated in the case of air, or its constituent gases, up to frequencies here employed, viz. 800 kc./sec. Experimental evidence with regard to these gases would have to be sought at higher temperatures or at higher frequencies. The work of Kneser has received much experimental and theoretical support from Henry(21), E. Grossman(22), and, less directly, from G. G. Sherratt and E. Griffiths(23). The present author gives additional data and confirms the results obtained by the various workers with an acoustical interferometer by a method which is more direct in that λ is obtained free from many of the causes of multiple peaks and associated complexities. Thus, in the above experiment the tube was always adjusted for resonance and so did not induce in the crystal a parasitic frequency of oscillation such as that observed by Hershberger, when the moving reflector altered the acoustical load on the crystal. In the present experiments it was found that such parasitic oscillations could be induced in certain crystals by throwing the tube out of resonance, i.e. the 219·3-kc./sec. crystal would oscillate at a frequency of 201 kc./sec.

Comparatively few experiments seem to have been made on the velocity of sound in nitrogen at supersonic frequencies. Using their resonance-tube method at sonic frequencies Shilling and Partington gave a value which at 0° C. becomes 337·4 m./sec., while J. A. Van Lammeren and W. H. Keesom(24), also using a resonance-tube method, obtained values of 337·1 and 337·5 m./sec. It may be noted that the latter author's value for air obtained with the same method is 332·1 m./sec.

Shilling and Partington made observations on oxygen at some frequencies and obtained a value of the velocity equal to 314·4 m./sec. Keesom, Itterbeck and Lammeren(25), also using a resonance-tube method, gave 315·4, while Pielemeier, using incomplete satellite values from his interferometer, gave a value in close agreement with that predicted by theory, namely 314·8 m./sec. This latter value is also in very good agreement with that obtained by E. G. Richardson(26), for the average of his results between frequencies of 42 and 1800 kc./sec. is 314·9 m./sec.

§ 8. ACKNOWLEDGEMENT

In conclusion I wish to express my sincere thanks to Prof. E. N. da C. Andrade, F.R.S., who not only formulated the problem but continued to show great interest in the progress of the work and offered many valuable suggestions during the course of it.

REFERENCES

(1) Pearson. *Proc. phys. Soc.* **47**, 136 (1935).
(2) Andrade. *Philos. Trans.* A, **230**, 413 (1932).
(3) Wood, A. B. *Proc. phys. Soc.* **47**, 145 (1935), Discussion.
(4) Wood, A. B. *Sound*, p. 264. (London, 1930.)
(5) Whytlaw-Grey and Patterson. *Smoke* (1932).
(6) Parker, R. C. *Trans. Faraday Soc.* **32**, 1115 (1936).
(7) Pierce. *Proc. Amer. Acad. Arts Sci.* **60**, 284 (1925).
(8) Hubbard. *Phys. Rev.* **36**, 1668 (1930).
(9) Pielemeier. *Phys. Rev.* **36**, 1005 (1930); **38**, 1236 (1931).
(10) Reid. *Phys. Rev.* **35**, 814 (1930); **37**, 1147 (1931).
(11) Kao. *Ann. Phys.*, Paris, **17**, 315 (1932).
(12) Hershberger. *Physics*, **2**, 269 (1932).
(13) Grabau. *J. acoust. Soc. Amer.* **5**, 1 (1933).
(14) Norton. *J. acoust. Soc. Amer.* **7**, 16 (1935).
(15) Shilling and Partington. *Phil. Mag.* **6**, 920 (1928).
(16) Cornish and Eastman. *Phys. Rev.* **33**, 90, 258 (1929).
(17) Vance. *Phys. Rev.* **39**, 737 (1932).
(18) Kaye and Sherratt. *Proc. roy. Soc.* **141**, 123 (1933).
(19) Ishii. *Sci. Pap. Inst. phys. chem. Res.*, Tokyo, **560**, 201 (1935).
(20) Kneser. *Ann. Phys.*, Lpz., **11**, 777 (1931).
(21) Henry. *Proc. Camb. phil. Soc.* **28**, 249 (1932).
(22) Grossman. *Ann. Phys.*, Lpz., **13**, 681 (1932).
(23) Sherratt and Griffiths. *Proc. roy. Soc.* A, **147**, 292 (1934); **156**, 504 (1936).
(24) Lammeren and Keesom. *Proc. K. Akad. Wet. Amst.* **35**, 6, 727 (1932).
(25) Keesom, Itterbeck and Lammeren. *Proc. K. Akad. Wet. Amst.* **34**, 7, 996 (1931).
(26) Richardson. *Proc. roy. Soc.* **146**, 56 (1934).

PHOTOMETRIC TEST PLATES

By A. K. TAYLOR, A.C.G.I., M.I.E.E.

Received 19 *October* 1936. *Read* 3 *December* 1936

ABSTRACT. Characteristic curves showing the variation of reflection factor with the angles of incidence and view are given for flat plates of seven materials. Suggestions are then made as to the use of these either as substandards of reflection factor or for the measurement of illumination.

§ 1. INTRODUCTION

Photometric test plates are required for use (*a*) with illuminometers in the measurement of brightness or illumination, and (*b*) as substandards of reflection factor. For the former purpose the test plate should have a plane matt surface with a constant apparent reflection factor* at some definite angle of view for all angles of incidence. The surface should be easy to clean without affecting its reflection characteristics, or else easy to reproduce. For the second purpose the test plate should have a plane matt surface giving a definite apparent reflection factor for certain specified angles of view and incidence. The surface must be such that its reflection factor for the conditions specified can be readily reproduced with a variation not exceeding 0·5 per cent; and this reflection factor must remain constant for a reasonable period.

Test plates made of the following materials were examined (1) white pot opal glass (ground), (2) magnesium oxide, (3) white filter paper, (4) white matt celluloid, (5) magnesium oxychloride, (6) white blotting paper, and (7) plaster of Paris.

§ 2. METHOD OF MEASUREMENT

The relative values of the apparent reflection factors were obtained by taking, with an illuminometer, readings of the brightness of the test surfaces at various angles of incidence when illuminated by the source at a constant distance. These readings, when divided by the cosines of the respective angles of incidence, were then proportional to the brightnesses of the test surfaces at those angles for an illumination numerically equal to that produced by the source at normal incidence.

The source used was an ordinary electric photometric substandard lamp of the grid type, the horizontal candle power being about 40 and the length of the grid 7 cm. The distance from the source to the test surface was in every case at least 10 times the length of the filament.

---

* The apparent reflection factor of a surface is the ratio of the brightness of the surface to that of a perfect diffuser of unit total reflection factor under equal illumination. The total reflection factor is the ratio of the total reflected light flux to the incident flux.

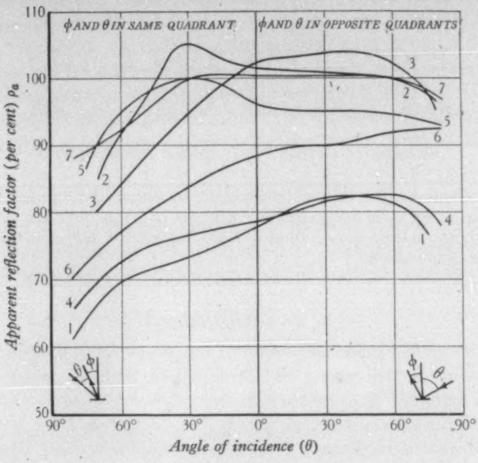


Figure 1. Variation of apparent reflection factor with angle of incidence at most favourable angle of view (see table 1). (Angle of view in plane of incidence.)

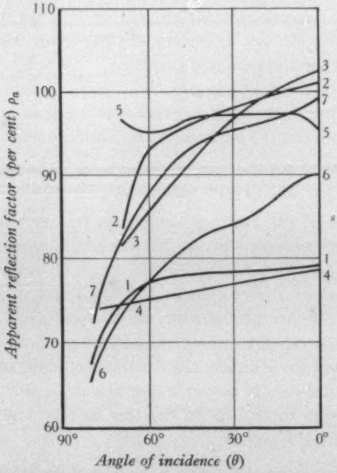


Figure 2. Variation of apparent reflection factor with angle of incidence at most favourable angle of view (see table 1). (Plane containing angle of view normal to plane of incidence.)

§ 3. RESULTS

Figures 1–4 show, for the seven materials examined, the variation in the apparent reflection factor ρ_a with the angle of incidence θ for a definite angle of view ϕ, the angles of view and incidence being in the same plane (figure 1) or in planes normal to each other (figure 2). The curves in figures 3 and 4 show the variation in the apparent reflection factor with the angle of view for normal incidence and for 45° incidence respectively.

The angles of view selected for the curves shown in figures 1 and 2 are those for which the changes in ρ_a corresponding with the changes in the angle of incidence were found to be a minimum for the material under consideration, where the angles of incidence and view were in the same plane but in opposite quadrants. It was found generally that large changes in the angle of view had very little effect on the value of ρ_a where the angles of incidence and view were in planes normal to each other.

The values of the total reflection factor ρ for a parallel beam at normal incidence, obtained by graphical integration[*] of the curves in figure 3, are given in table 1, which also shows for each material the amount by which ρ_a changes, expressed as a percentage of the mean value of ρ over the range of θ covered.

Table 1. Reflection factors and their variation

Material	ρ (per cent)	θ and ϕ in ...	Change in ρ_a with direction of incidence (%)					
			same plane				perpendicular planes	
		ϕ \ θ	0–80°		10–70°		10–70°	
			Max.	Mean	Max.	Mean	Max.	Mean
Pot opal glass	76	30°	4	2½	3	2	2¾	2
Magnesium oxide	98	40°	6	1¼	1½	¾	6½	4¾
White filter paper	96	20°	7	2	3	1½	12	6
White matt celluloid	—	30°	2½	1½	2	1¼	2½	1¾
Magnesium oxychloride	94	40°	2	1	1	¾	2	1
White blotting paper	93	20°	2¼	1½	2	1½	10	5
Plaster of Paris	97	30°	2	1	1½	¾	10	4½

---

[*] These total reflection factors have been deduced graphically from the curves for normally incident light, on the assumption that under these circumstances the brightness of the surface depends only on the angle of view. If F is the total flux incident on the surface, that leaving the surface in a direction at an angle ϕ with the normal is $F\pi^{-1}\rho_\phi \cos\phi$, and since the corresponding element of solid angle is $2\pi \sin\phi \, d\phi$ the total reflected flux is $F\int_0^{\pi/2} \rho_\phi \sin 2\phi \, d\phi$. The reflection factor is obtained by giving F the value unity, and is thus $\int_0^{\pi/2} \rho_\phi \sin 2\phi \, d\phi$. From the curves of $\rho\phi$ in figure 3 curves representing $\rho_\phi \sin 2\phi$ were plotted, and the integrals found by measuring the areas enclosed between curve and base.

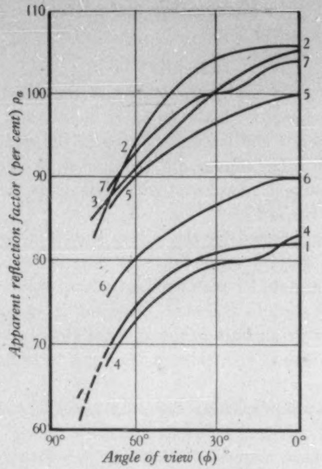


Figure 3. Variation of apparent reflection factor with angle of view for normal incidence.

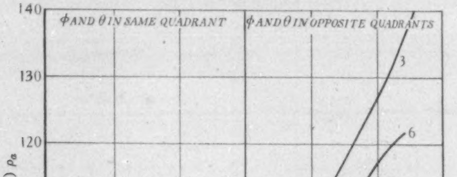


Figure 4. Variation of apparent reflection factor with angle of view with light incident at 45° in plane containing angle of view.

§4. EXPERIMENTAL DETAILS

Pot opal glass. A sample of good colourless commercial pot opal glass, about $2\frac{1}{2}$ mm. thick, ground to a medium matt surface, was mounted in a frame with a piece of black velvet in contact with the back of the sample. This surface is suitable for use when making measurements of illumination provided the angle of incidence be limited to 70°, ϕ and θ being in the same plane but in opposite quadrants, or in planes normal to each other. This surface is also suitable as a substandard of reflection factor. It is suggested that the disk should be calibrated at such an angle of incidence that the mean value of ρ_a is obtained.

Magnesium oxide. A considerable amount of work has been carried out on magnesium oxide(1). It was tested in the form of a flat surface upon which the oxide from burning magnesium ribbon had been smoked. It was found that unless this oxide covering was thick the original polished copper surface absorbed an appreciable amount of light. To prevent this, the surface was silver-plated before depositing the oxide; then, provided that the oxide coating was thicker than about $\frac{1}{3}$ mm., the absorption by the backing was inappreciable. Under these conditions the value of the total reflection factor ρ for diffuse incidence was found to be reproducible within 0·5 per cent for successive fresh coatings of the oxide using the same stock of ribbon.

This surface is hardly suitable for use in the measurement of illumination on account of its fragility. With regard to its use as a substandard of reflection factor, it has been found that if it is protected from dust the total diffuse reflection factor ρ for diffuse incidence will remain constant for 3 weeks within 0·15 per cent. If left for a longer period there may be a change due to the formation of magnesium carbonate. This matter has not been fully investigated, though exposure of the surface to damp CO_2 for several periods of 12 hours produced no definite change. An analysis of old coatings of MgO gave 5 per cent of carbonate.

Filter paper. Samples of best white filter paper (Muncktell's No. 0) were used, ten pieces in one pack being used to form a suitable disk 9 cm. in diameter. For the measurement of illumination this material may be used where great precision is not required, as it is usually obtainable without difficulty. As regards its use as a substandard of reflection factor, it seems likely that with clean samples the value of ρ or ρ_a would not vary by as much as $\frac{1}{2}$ per cent, provided that at least nine sheets were used in the disk. It has also been found that prolonged exposure to ultra-violet radiation does not affect the value of the reflection factor.

Matt celluloid. Samples of matt celluloid, specially sand-blasted and flattened, were obtained from one of the manufacturers of illumination photometers. When this material is used in the measurement of illumination there is a difficulty in keeping the surface flat. Moreover, it tends to become discoloured with exposure to light, and it cannot be cleaned easily without its reflection characteristics being affected.

Magnesium oxychloride. Plates of magnesium oxychloride were prepared by Mr J. S. Preston of the National Physical Laboratory by mixing magnesium oxide with a strong solution of magnesium chloride and allowing the resultant paste to set between glass plates covered with a thin film of grease to prevent sticking.* The plates used were about 2–3 mm. thick and were mechanically strong. A matt surface could be produced by grinding with carborundum powder, though care had to be taken to soak the disks thoroughly in water beforehand to prevent them from sticking to the grinding plate. Two samples were made, sample A comprising MgO 10 g., $MgCl_2.6H_2O$ 10 g., while sample B comprised MgO 10 g. and $MgCl_2.6H_2O$ 7 g.

The values of the total diffuse reflection factor for diffuse incidence were for A 0·955 and for B 0·960.

This material seems to be suitable for use in the measurement of illumination, especially for street lighting when it is necessary to measure the illumination between two lamps due to both sources. There is, however, some difficulty in cleaning the disks, so that if they are used for outdoor work care must be taken to prevent dust and moisture from reaching them. As a substandard of reflection factor this material may be useful if it can be shown that there is no appreciable change in composition with time.

Blotting paper. Samples of good white blotting paper (Ford 428) 0·6 mm. thick were flattened and mounted. The use of this material in the measurement of illumination or as a substandard of reflection factor is inadvisable owing to probable variation from sample to sample. Further it was found that a distinct discoloration was produced by one week's exposure to ultra-violet radiation from a carbon arc.

Plaster of Paris. Samples of plaster of Paris were made up with as flat a surface as possible. This material is not suitable for illumination measurements or as a substandard of reflection factor on account of the difficulties encountered in cleaning.

Selectivity. Samples of magnesium oxide, magnesium oxychloride and of plaster of Paris were tested at three regions of the spectrum corresponding approximately to 6500, 5400 and 4700 A.

The values obtained are given below:

Total diffuse reflection factor

	White light	6500 A.	5400 A.	4700 A.
Magnesium oxide				
Before exposure to air	0·973	0·973	0·973	0·972
After exposure to air	0·972	0·973	0·971	0·966
Magnesium oxychloride	0·960	0·972	0·956	0·934
Plaster of Paris	0·953	0·972	0·952	0·964

REFERENCES

(1) Preston, J. S. *Trans. opt. Soc.* **31**, 15 (1929–30).
(2) Preston, J. S. *J. sci. Instrum.* **12**, 197 (1935).

DISCUSSION

J. S. Preston. This paper is particularly interesting to those concerned with photometry in that it gives extensive information about the properties of a number of materials of possible use as test plates, whose brightness by reflected light is commonly measured with some such visual photometer as the Macbeth illuminometer. It should be interesting and instructive to compare these properties with those of translucent diffusing materials such as may be used in front of a photocell in a photoelectric illumination-meter. An examination of such translucent materials and their brightness by transmitted light would seem to offer a useful field of research.

Two points have interested me in particular in the author's results. The first arises in consideration of figure 1, in which it is seen that curves 2 and 5 are alike in showing a high value of reflection factor for angles of view in the neighbourhood of the direction of incidence, rather than at the specular angle. It is curious that in both the test plates concerned magnesium oxide is present, and perhaps the author could say whether there may be anything inherent in the fine structure of the oxide which may account for the shape of the curves and for their similarity.

Secondly, on reference to table 1 it is seen that while the behaviour of magnesium oxychloride is similar to that of plaster of Paris for observations made in the plane of incidence, it is not so for observations in the plane at right angles, in which case the brightness variations are much greater for the plaster of Paris. It would be interesting to know whether the better behaviour of the oxychloride might be due to deeper penetration of the light into the body of the diffuser, in view of the general similarity of the two materials in other respects.

Author's reply. Mr Preston's remarks certainly suggest a partial explanation of the behaviour of plates 2 and 5. As he has himself had considerable experience in making up samples containing magnesium oxide it seems probable that he is right.

I also agree that the smaller variations in the apparent reflection factor of the magnesium oxychloride plate might be explained in the way he suggests.

THE MEASUREMENT OF DISTORTION IN A GAUGE PROJECTION LENS

By JOHN S. ANDERSON, M.A., D.Sc., F.Inst.P.,
Optics Division, The National Physical Laboratory

Received 24 *June* 1936. *Read* 3 *December* 1936

ABSTRACT. The distortion of a lens working at finite magnification is deduced from measurements of the variation of magnification of an object as the latter is traversed across the object field or the image field. It is found that as regards attainable accuracy there is little to choose between the alternative methods of measuring an enlarged image of an object in the object field or a reduced image of an object in the image field. The technique of the method as applied to a new gauge projection lens is described.

---

§ 1. INTRODUCTION

The measurement of distortion in a lens designed for use with parallel incident light is, comparatively speaking, a simple matter. The usual method is to employ a nodal slide bench and to measure the displacements of the image of a distant object as the lens is rotated about an axis, perpendicular to the lens axis, passing through the back nodal point. With lenses designed for use at other magnifications than zero the problem is simple in theory, but the experimental procedure is more complicated. If the object and image fields are both reasonably small the normal method is first to determine the conjugate planes corresponding to the magnification required, and then to move a suitable object, such as cross-lines, to definite positions in the object plane and measure the co-ordinates of the image positions in the image plane. The distortion at any point of the field is then the displacement of the image from the position it would occupy if the magnification given by the lens were constant over the whole of these conjugate planes. If, however, either the object or the image field is comparatively large, this direct method may be difficult to carry out, for a high degree of accuracy is usually required. In these circumstances it is simpler to deduce the magnitude of the distortion from measurements of the variation of the enlargement or reduction of an object of appreciable length placed at different positions in one of the conjugate planes. The length of the object can be chosen so that the size of the image comes within the range of the most accurate instruments available for the measurements.

It often happens that when this method is used there is no marked disparity in the size of object and image. For instance, measurements at magnifications of unity and one half are not uncommon, and one quarter occurs occasionally. For these ratios it is mainly a matter of convenience whether the object of constant length is placed in the plane where the size is the larger or the smaller of the two related values. When the magnification is large, some doubt may arise whether it is better to measure variations in size on an enlarged image, where they may be expected to be measurable without refined apparatus, or on the other hand to use a large object and make fine measurements over short lengths with apparatus of high accuracy. No evidence on this point, which is both of practical importance and of theoretical interest, appears to have been published. There also appears to be a lack of a description of the experimental procedure suitable for application in such circumstances. It is therefore thought that an account based on tests recently carried out at the National Physical Laboratory on a new gauge projection lens may be of some general interest. Incidentally the results obtained indicate the order of magnitude of the residual distortion to be expected in an optical system where the removal of distortion has received special consideration.

The lens submitted was a Taylor-Hobson profile projection lens (focal length 10 in.) designed to project accurate images of objects up to 4 in. in diameter at a linear magnification of 25. As the lens gave satisfactory definition over the whole field, the main object of the test was to measure the distortion. The question of enlargement or reduction arose immediately. This was decided in the first place by a special property of the lens. In most optical systems the central ray of the effective beam of light proceeding from a point at a distance from the axis is about equally inclined to the axis on both sides of the lens. In a system intended for projecting profiles, however, it is important to have the central rays on the side where the object is placed very approximately parallel to the axis over the whole effective field. This implies that in a test of this kind of system a travelling microscope can be used very conveniently for examining and measuring a reduced image, for the displacements do not exceed the range over which accurate measurements can be made on the screw of the instrument, and the microscope itself will be correctly disposed to receive the light from the lens under test symmetrically when it is held in its normal position perpendicular to the axis of the screw. These factors led to a calibration of the lens based on measurements of an image approximately one twenty-fifth the size of the object used. It was subsequently suggested that it would be of interest to compare the results obtained on reduction with those derived on enlargement, the way in which the lens is normally used. Two series of measurements were made under these conditions, and one of these is given here to show how the two methods of calibration compare with one another.

§ 2. EXPERIMENTAL PROCEDURE

The set-up adopted for the first test is shown diagrammatically in figure 1, where *AB* represents the plane in which the gauge or other object is placed, *CD* the plane of the screen on which the image is projected, and *OP* represents the optical axis of the lens. A scale *EF*, consisting of vertical black lines on a clear background, was mounted so that it could be traversed along a horizontal 6-ft. steel optical bench, the length of the scale being parallel to the bench. The end-lines of the scale were illuminated by two lamps *L*, *L* placed behind the plane *CD*. A travelling microscope *M* was mounted so that in all positions of its run it was focussed on the plane *AB*. Adjustments of height and level were made to ensure that the plane traversed by the axis of the microscope was the same as that traversed by a line on the object scale perpendicular to the scale divisions. Both the bench and the travelling microscope were adjusted so that the optical axis of the projection lens was normal to the directions of motion of the scale and the microscope. This was done in the following way. A small illuminated pinhole was placed at *O* and a plane mirror was held against the lens mount on the side remote from *O*. The pinhole was then adjusted until its reflected image, formed by light which had passed twice through the lens, coincided with it. The pinhole was then on the optical axis of the lens, assuming that the optical and mechanical axes were coincident. This assumption was verified by noting that, on removing the plane mirror and looking along the lens axis, the images of the pinhole formed by reflection in the surfaces of the component lenses were collinear. This series of images thus defined the optical axis of the lens. The mirror was then mounted parallel to the optical bench and the mirror and bench were moved together until the series of images reflected by the mirror was collinear with the original series. The bench and scale were therefore then normal to the optical axis of the lens. The process was repeated with the illuminated pinhole on the optical axis at the other side of the lens and the mirror mounted parallel to the slide of the travelling microscope. This ensured that the focal plane of the microscope, that is the object plane *AB*, was normal to the optical axis.

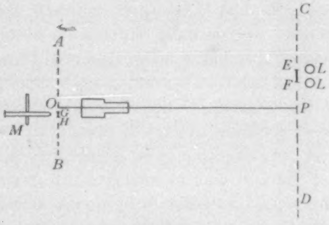


Figure 1.

A length of 100 mm. on the scale was used as the object *EF*, and the dimensions of its images *GH* in the plane *AB* were measured on the travelling microscope for different positions of the scale *EF* in the plane *CD*. Distances were measured from the points *O* and *P* where these planes were intersected by the optical axis of the lens.

The second test was effectively the first one reversed. Instead of measuring the reduced image of an object in the plane *CD*, the enlarged image of an object in the plane *AB* was measured for different positions of the object. A scale, consisting of transparent vertical lines on a dark background, was mounted so that it could be traversed in the object plane *AB*, and the base and carriage of the travelling microscope were set up behind the plane *CD*. As the object was magnified 25 times by the projection lens, it was found desirable to replace the microscope by a low-power magnifying lens focussed on a suitable scale. This change allowed measurements to be made even at the corners of the field without any alteration in the direction of the magnifier.

Adjustments similar to those of the first test were made to ensure that the optical axis of the projection lens was normal to the plane of motion of the object scale and to the plane traversed by the scale of the magnifier. A length of 4·064 mm. on the object scale was used and the dimensions of its images were measured on the screw of the travelling microscope for different positions of the scale in the object plane.

§ 3. OBSERVATIONS

The measurements made in the first test are given in table 1. The first column gives the distance of the centre of the scale from the axis of the system, the second column the length of the image, and the third column the difference between the length of the image and that of the image at the centre of the field.

Table 1. (Length of object 100·00 mm.)

Distance of centre of scale from axis (m.)	Length of image (mm.)	Difference (mm.)
−0·85	3·9850	−0·0035
−0·75	3·9855	−0·003
−0·65	3·9835	−0·005
−0·55	3·9845	−0·004
−0·45	3·9840	−0·0045
−0·35	3·9865	−0·002
−0·25	3·9875	−0·001
−0·15	3·9885	0
−0·05	3·9885	0
0	3·9885	0
+0·05	3·9880	−0·0005
+0·15	3·9880	−0·0005
+0·25	3·9875	−0·001
+0·35	3·9860	−0·0025
+0·45	3·9860	−0·0025
+0·55	3·9850	−0·0035
+0·65	3·9845	−0·004
+0·75	3·9860	−0·0025
+0·85	3·9895	+0·001

The measurements made in the second test are given in table 2. The first column gives the distance of the centre of the image of the scale in the image plane from the axis, the second column the length of the image, the third column the length of the object which would give at each part of the field an image equal in length to the image at the centre of the field, and the fourth column the difference between the length of the object and that of the object at the centre of the field. The results are thus put in the same form as those obtained in the first test.

Table 2. (Length of object 4·064 mm.)

Distance of centre of image of scale from axis (m.)	Length of image (mm.)	Length of object giving image of length 103·595 mm. (mm.)	Difference (mm.)
−0·845	103·595	4·064	0
−0·80	103·639	4·062	−0·002
−0·60	103·725	4·059	−0·005
−0·40	103·655	4·062	−0·002
−0·20	103·626	4·063	−0·001
0	103·595	4·064	0
+0·20	103·625	4·063	−0·001
+0·40	103·650	4·062	−0·002
+0·60	103·683	4·060	−0·004
+0·773	103·675	4·061	−0·003

§4. ANALYSIS OF OBSERVATIONS

To determine the reliability and consistency of the observations, a formula was fitted to the observations given in the first two columns of table 1 by the method of least squares. Preliminary examination of the figures indicated that there was no distinct evidence that more than the first two orders of distortional aberration were present, together with a uniform change of scale on proceeding from one end of the image to the other. This change of scale may be ascribed to a very slight inclination of the image plane to the position it should ideally occupy. The curve corresponding to the formula is plotted in figure 2, where the small circles indicate the observations recorded in the second column of table 1. It will be seen that the fit is satisfactory. From this formula an expression was derived (as shown later) giving the distance of a point of the gauge or other object from the axis in terms of the corresponding distance (approximately twenty-five times greater) of the image point. If x denotes the distance of the object point from the axis in millimetres, and X the distance of the corresponding image point from the axis in metres, the equation obtained is

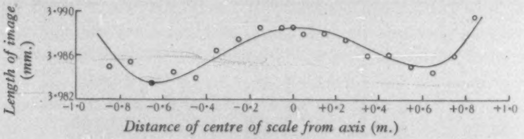


Figure 2.

$$x = 39{\cdot}88528X + 0{\cdot}05934X^2 - 0{\cdot}07187X^3 + 0{\cdot}05526X^5.$$

This formula was also used with the omission of the term involving X^2 (which corresponds to the error in normality of the object and image planes in the first test) to construct figure 3, on which are shown the observations given in table 2. It will be seen that these observations fit quite as well as those from which the formula was derived. The diagram (also other observations which are not recorded here) shows that we may regard this theoretical curve as a correct representation of the properties of the instrument. Particulars of the calibration computed from the formula just given are recorded in table 3.

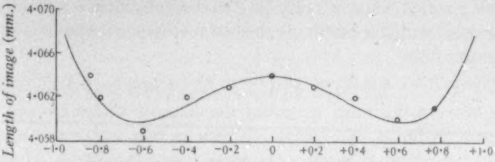


Figure 3.

Table 3

Object distance in image plane (m.)	Image distance in object plane (mm.)	Displacements towards axis represented by third and fifth order terms (mm.)	Magnification	Compensated* displacements (mm.)
0	0	0	0·0398853	0
0·05	1·99426	0·00001	0·0398851	−0·00093
0·10	3·98846	0·00007	0·0398846	−0·00180
0·15	5·98255	0·00024	0·0398837	−0·00257
0·20	7·97650	0·00056	0·0398825	−0·00318
0·25	9·97025	0·00107	0·0398810	−0·00360
0·30	11·96378	0·00181	0·0398793	−0·00380
0·35	13·95706	0·00279	0·0398773	−0·00375
0·40	15·95008	0·00403	0·0398752	−0·00344
0·45	17·94285	0·00553	0·0398730	−0·00288
0·50	19·93538	0·00726	0·0398708	−0·00209
0·55	21·92773	0·00918	0·0398686	−0·00110
0·60	23·91994	0·01123	0·0398666	+0·00001
0·65	25·91211	0·01333	0·0398648	+0·00118
0·70	27·90433	0·01536	0·0398633	+0·00228
0·75	29·89675	0·01721	0·0398623	+0·00319
0·80	31·88953	0·01869	0·0398619	+0·00374
0·85	33·88287	0·01962	0·0398622	+0·00373
0·90	35·87699	0·01976	0·0398633	+0·00294
0·95	37·87216	0·01886	0·0398654	+0·00110
1·00	39·86867	0·01661	0·0398687	−0·00208

* A positive sign indicates a displacement towards the axis of the system.

The first column gives the distance of an object point on either side of the axis in metres, the second gives the corresponding image distance in millimetres, the third gives the displacements represented by the third and fifth order terms in the expression for x, and the fourth shows the variations in the magnification. From the third column it will be seen that the maximum displacement corresponding to the aberrational terms is almost 1/50 mm., that is 8 ten-thousandths of an inch.

Analysis of the distortion in this manner, though agreeing with that followed in theoretical discussions, does not correspond most closely to what is required in practice. It attaches very great importance to a quantity which in application is not by itself significant—the limit to which the magnification tends as the object considered shrinks to an infinitesimal length centered on the lens axis. Instead of this representation we require to know how far an image point may be displaced from its ideal position when we take for the magnification a mean value for the size of field considered. To determine this we write the equation between x and X in the alternative form

$$x=39{\cdot}86659X+0{\cdot}00345375\,[16X^5-20X^3\times 1{\cdot}04046+5X\times(1{\cdot}04046)^2].$$

As X varies between the limits $\pm 1{\cdot}02003$ the quantity within the square brackets does not exceed the limits $\pm(1{\cdot}02003)^5$. If then we take as a reference standard the magnification 0·03986659 instead of the axial value 0·03988528, we find that the displacement of a point of the image from its computed position lies within the limits ±0·004 mm. or ±1·6 ten-thousandths of an inch. The calculated displacements corresponding to this magnification are given in the fifth column of table 3. The points for which the displacement is zero are

$$X=0, \quad X=\pm 0{\cdot}59956, \quad X=\pm 0{\cdot}97011.$$

This suggests that the magnification could advantageously be determined by setting a symmetrically placed object 48 mm. in length to project as 1·2 m. on the screen.

The relation between the two methods of presenting the results of the calibration is shown in figure 4. The curved line shows the relation between the third and fifth order terms in the expression for x as ordinates and X as abscissae. By introducing a suitable first order term also, and thus changing the nominal magnification by a small amount, we change the axis from which ordinates are to be measured from the horizontal line, which is tangential to the curve at its centre, to the inclined straight line which cuts the curve in five points. These ordinates have the values described as compensated displacements in the fifth column of table 3.

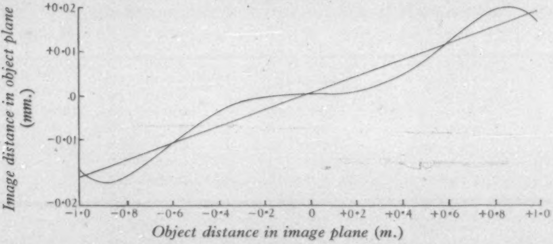


Figure 4.

§ 5. THEORY

It may be helpful to include a word on the way in which the equation for x is to be found. Suppose that an object of length $2L$ is measured, and its image is found to be of length $2l$ when the centre of the object has the co-ordinate X. If l is ascertained for several values of X, we can in a variety of ways, e.g. least squares, derive a formula

$$l=\sum_{r=0}^{m} a_r \frac{X^r}{r!},$$

which represents the observations satisfactorily. The formula we wish to construct is

$$x=f(X),$$

which is connected with the observations by the relation

$$2l=f(X+L)-f(X-L).$$

Evidently f is of order $m+1$ in X. If $f_n(X)$ denotes $\dfrac{d^n f(X)}{dX^n}$, and F_n stands for $f_n(0)$, we have

$$2l=\sum_{r=0}^{m} \frac{X^r}{r!}\left[f_r(L)-f_r(-L)\right],$$

so that

$$a_r=\sum_{n=0} \frac{L^{2n+1}}{(2n+1)!} F_{2n+r+1}.$$

The solution of these equations is

$$LF_{2n}=a_{2n-1}+\sum_{r=0} \frac{(-)^r (2^{2r}-2) L^{2r} B_r}{(2r)!} a_{2n+2r-1},$$

$$LF_{2n+1}=a_{2n}+\sum_{r=0} \frac{(-)^r (2^{2r}-2) L^{2r} B_r}{(2r)!} a_{2n+2r},$$

where B_r denotes the rth Bernoullian number. The first seven of these numbers are

$$B_1=\tfrac{1}{6},\quad B_2=\tfrac{1}{30},\quad B_3=\tfrac{1}{42},\quad B_4=\tfrac{1}{30},\quad B_5=\tfrac{5}{66},\quad B_6=\tfrac{691}{2730},\quad B_7=\tfrac{7}{6}.$$

It will be noted that F_0 is not determinable from this theory. It is an arbitrary constant of integration, and in this particular problem is given the value zero so that x and X vanish together.

§ 6. CONCLUSIONS

From the analysis of the observations it may be concluded that the projection lens will enable measurements to be made with an error not exceeding ±0·004 mm., or ±1·6 ten-thousandths of an inch as measured in the object plane. It also appears that, under the special conditions of this particular test at all events, there is little to choose, as regards attainable accuracy, between the two alternative methods of carrying out the test. The second set-up, in which an enlarged image of the object is formed and examined, should be the better one to employ, as the error in measuring the longer image is, with a given instrument, relatively less. It suffers, however, from the disadvantage that, as the travelling microscope has to be moved from point to point in the image field, it is more difficult to attain the requisite degree of rigidity than in the alternative method. A further disadvantage is that the test takes longer to carry out as longer distances have to be traversed by the travelling microscope. This difficulty could, of course, be overcome by using two magnifying lenses for setting on the end marks of the scale, one being fixed to the stationary portion of the travelling microscope and the other to the movable portion.

§7. ACKNOWLEDGEMENTS

I am indebted to Messrs Taylor, Taylor and Hobson, Ltd., of Leicester, the makers of the projection lens, for permission to publish these results, and to Mr T. Smith, M.A., F.R.S., for the analysis of the observations.

DISCUSSION

A. **Warmisham.** On behalf of Taylor, Taylor & Hobson, Ltd., I must acknowledge, with thanks, the excellent co-operation of the National Physical Laboratory in these measurements. Our practice is to measure distortion in the screen plane: we use a recently constructed projector in which the difficulties of this method, including those alluded to by the author, appear to have been overcome. The original design of the lens referred to in the paper is by Mr H. W. Lee.

J. W. **Perry.** In considering the term in X^2 in § 4 one should, I think, carefully distinguish what is primarily being tested—whether it is the formula of which the lens is a realization or the particular lens as an example of a symmetrical optical instrument. The term in X^2 is of course justifiably rejected when attention is confined to the lens formula; but when testing the lens, agreement of the nature indicated in figure 3 only establishes a presumption, strong in this instance, in favour of such rejection. Assuming the first cause for such a term to be removed by perfect *mechanical* adjustment of the testing-apparatus, two further cases may arise to cause such a term, namely (*a*) disagreement of direction of mechanical and optical axes, and (*b*) a constructional defect of asymmetry in the lens. The author has taken account of case (*a*) and I do not suggest that case (*b*) applies in this instance, but in considering the test as such the full interpretation of the X^2 term needs, I think, to be borne in mind when lenses of such a high degree of correction are being tested.

R. **Kingslake.** In connexion with the measurement of distortion in photographic lenses, I should like to ask the author how he would define distortion when coma is present. Is it to be regarded as a transverse displacement of the principal ray, or a displacement of some kind of centroid of the comatic light patch? The first is the usual theoretical conception of distortion, yet the second would appear more reasonable.

Author's reply. I agree with Mr Perry's remarks. In the lens under discussion there was, however, no indication of the presence of a defect of either type (*a*) or type (*b*).

With regard to Prof. Kingslake's remarks I think that Mr Smith is more qualified to supply an answer.

Mr T. **Smith.** As the author has appealed for my opinion on Prof. Kingslake's question, I may say that I think it important to recognize that what we do in experimental work may not correspond with any particularly simple theoretical case. If it were important to determine the theoretical setting with appreciable coma present, the best course would be to make a careful analysis of observations on an instrument of very accurately known construction.

THE STRUCTURE AND ELECTRICAL CONDUCTIVITY OF THIN FILMS OF INDIUM

By V. E. COSSLETT,
H. H. Wills Laboratory, University of Bristol

Received 1 *November* 1936. *Read* 11 *December* 1936

ABSTRACT. The high electrical resistance and negative temperature coefficient that thin films of indium, in common with other metals, exhibit when prepared under moderate vacuum conditions (10^{-4} to 10^{-5} mm.) and without exhaustive cleaning of the substrate, are shown by electron-diffraction methods to be in no way connected with the existence of any amorphous form of the metal. The lattice of freshly deposited films is found to correspond in form with that of the bulk metal but to have a spacing 0·7 per cent greater than that given by recent X-ray work, gold films here being taken as standards. With slowly deposited films the spacing increases and the axial ratio decreases with rise in temperature up to room conditions, and exposure to air carries this process further until a film showing a cubic indium lattice with spacing 3·4 per cent greater than the bulk metal is obtained. This change is accompanied by slow oxidation, the oxide lattice being 0·6 per cent larger than X-ray values indicate. It is suggested that the high resistance of these films is due to sorption of residual gas, and that the negative temperature coefficient is the result of desorption and resorption processes; changes in crystal-size with temperature are also noted and discussed. The change in lattice constants is ascribed to the action of gas within the lattice, possibly amounting to the production of intermediate stages in oxide-formation.

§ 1. INTRODUCTION

A NUMBER of experimenters have found during the past forty years that very thin films of metals, whether chemically deposited(1) or sputtered(2) or produced by evaporation(3), have a very high electrical resistivity. Whatever the metal, this resistivity is greater by many orders than that of the metal in bulk and consequently a number of workers concluded that the properties of metals are essentially modified when they are formed into thin films. This conclusion has been upset by the recent work of Lovell(4) who has shown that the high resistivity is not a property of all states of the metal in thin films, but must be ascribed to disturbing factors not hitherto eliminated in their preparation. He succeeded in preparing stable films of rubidium, only a few atomic layers thick, that showed conductivities that departed from that of the bulk metal only by amounts to be expected on theoretical grounds. At the same time he was able to repeat some of the results obtained by previous investigators by particular treatment of the surface on which the films were formed.

Before the completion of his work a parallel investigation had been made by the writer on the structure of films of indium laid down by evaporation, the opportunity being taken to study their resistivity at the same time. It is unfortunate that the requirements of the determination of structure by electron-diffraction did not permit of the use of Lovell's high refinement. Nevertheless the nature of the structure of metallic films as normally obtained by evaporation in vacuo still remains a matter of interest and the following is a record of the results that have been obtained.

It is known(3, 5) that the degree of vacuum and cooling of the receiving surface have an important effect on the formation of the film, and that sputtered films may contain up to their own volume of gas(6). Care was taken, therefore, to control the rate of deposition of the film, the temperature of the surface, and the degree of vacuum. The best vacuum consistent with the structural determinations was 10^{-4} to 10^{-5} mm., and the evaporated metal was deposited on a surface cooled by liquid air. Indium was chosen as it lent itself to an accurate method of determining the density of the evaporated beam of metal atoms prior to condensation; uncertainty as to the thickness of the films formed has been one of the disadvantages in earlier work.

Some earlier investigators using X rays have claimed that thin metallic films are amorphous(7), others that they are of normal structure(8). Corresponding to these results, theories have been elaborated which ascribe the observed high resistances on the one hand to a special amorphous form of the metal(9), and on the other hand to lack of continuity in a normal crystalline structure, that is, to the formation of fissures or of isolated granules(10). The very much shorter wave-lengths employed in the electron beam, as compared with X rays, permit good spectra to be obtained with films less than 100 A. in thickness. Indeed, since Kirchner's work on evaporated films(11), several workers have obtained diffraction patterns from such films in the course of various investigations. Except in the case of antimony(12) these patterns have always corresponded to a normal structure.

No investigation had been made, however, of the structure of very thin films prepared under controlled conditions, nor of the possible correlation of structural change with electrical resistance. The results of the present work show that films of indium, from 10 to 100 atomic layers in thickness, have a crystal structure that differs only slightly from that of the pure bulk metal, and that slight structural changes can be observed to accompany changes in resistance under varying conditions. The relationship of these structural changes to the resistance-changes is not, however, in all respects clear.

§ 2. EXPERIMENTAL DETAILS

Preparation of the films. The indium was evaporated from a gun, *G*, of stainless steel supported on a quartz finger within which was the heating-coil and which was mounted eccentrically on a ground joint. The gun could thus be rotated to face either the cooled receiving surface or the filament of a collector which recorded the intensity of the evaporated beam. The collector *C* consisted of a tungsten strip, 1 mm. in width, facing a slit of length 1 cm. and width 1 cm. cut in a metal cylinder, 4 cm. long and 1 cm. in diameter, which surrounded it. The strip of tungsten, maintained at red heat, acted as collector filament at a positive potential of 240 V. with respect to the cylinder.

The action of this collector has been described fully elsewhere⁽¹³⁾. Briefly, the metal atoms which strike the filament are positively ionized (the ionizing potential of indium being lower than the work function of tungsten oxide) and are then collected by the cylinder. The resultant current between filament and collector is a measure of the rate at which atoms are reaching the filament. When the beam-intensity has been determined in this way, the gun may be rotated to face the cooled receiving surface. For instance, a current of 3·6 A. in the heating-coil produced a beam-intensity of $6{\cdot}8 . 10^{13}$ atoms per cm^2 per second at the receiving surface, corresponding to a current of 1·0 μA. in the collector circuit. Assuming the metal to be uniformly distributed with normal structure in the film and every incident atom to stick when the surface is cooled with liquid air, this beam would deposit four atomic layers per minute. In order to test these assumptions, the optical density of a typical film was determined and the thickness was evaluated from the formula of Goldschmidt⁽¹⁴⁾. The agreement between the two methods was satisfactory, but as the formulae for optical density are themselves only approximate, the values given in this work must be read as indicating an upper limit of film-thickness, liable to reduction if any considerable proportion of the atoms are reflected from the receiving surface.

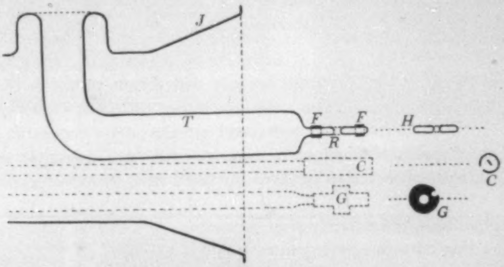


Figure 1. The evaporating system, showing the disposition of gun, collector and receiving surface *R*.

The end of a glass tube *T* was drawn out to a flat hollow tongue *H*, 1 × 2 cm., to which the receiving surface was secured by two clips of copper strip with screw fastenings, *F*, *F*. Liquid air within the tube served to cool the surface, the temperature of which was measured with a thermocouple. A small hole was drawn in the centre of the glass tongue to allow the electron beam free passage. When the receiving surface was easy to handle, for instance in glass or bakelite strips, silver was deposited at each end of it to make good contact with the copper strips. With cellulose film as substrate, two pieces of copper gauze were secured by the clips and the film was stretched between the gauzes. The cellulose film was from 100 to 300 A. thick, so that the deposited metal should overlap on to the gauze without discontinuity. The resistance of the metal film was measured directly, leads running from the copper clips to an external circuit.

The liquid-air vessel and the ground joints for the rotation of the gun and collector were all mounted on a large ground-glass joint *J* which rotated about a horizontal axis at the side of the electron-diffraction apparatus. This allowed the position of the receiving surface to be varied: during deposition it was at an angle of 45° to the horizontal, and during diffraction it was horizontal so that the electron beam could pass normally through the film.

Electron-diffraction apparatus. The apparatus used for producing the electron beam and registering the diffraction patterns was a modified form of that previously described(15), the main difference being the introduction of a focusing coil. The apparatus could be evacuated by fast pumps down to 10^{-5} mm., the electron beam entering it through a limiting diaphragm of diameter 0·1 mm.; a separate pumping system was used for the discharge tube which functioned with a cold cathode.

By using a camera box it was possible to take five photographs in succession without disturbing the vacuum. Gold films were used for calibrating purposes, the voltages employed being between 40 and 50 kV.

§ 3. RESULTS

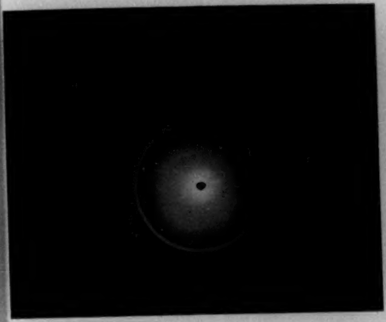
Determination of structure. Some seventy diffraction patterns were obtained from films of indium deposited on cellulose under different conditions; in each case the diameters of the rings were measured and the lattice constants evaluated in the usual way. Significant changes in ring-breadth and intensity, as well as in lattice constant, were observed in patterns obtained from films subjected to varying treatments.

Photographs taken immediately after deposition of a film at liquid-air temperatures showed a ring pattern agreeing in relative intensities with X-ray work on bulk indium, but giving a larger lattice spacing. The structure proved to be face-centred tetragonal for which $a = 4{\cdot}61 \pm 0{\cdot}005$ A. and the axial ratio $c/a = 1{\cdot}07 \pm 0{\cdot}005$. The early X-ray work of Hull & Davey(16) gave values of 4·58 A. and 1·06, and three recent determinations(17) agree in giving 4·583 A. and 1·077 for bulk indium at room temperature. During the preparation of this paper Riedmiller(18) has published results for nickel, gold and silver films, which also show a slightly increased lattice spacing of the same order of magnitude.

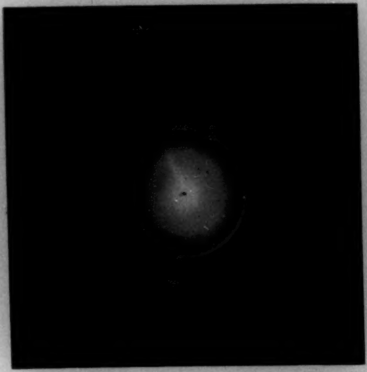
On allowing the indium films to rise to room temperature a slight change, outside the experimental error, occurred in the pattern, without alteration in the relative intensities of the rings; the constants now found were 4·62 A. and 1·06. Typical photographs taken at liquid-air and room temperatures are reproduced in the plate at A and B, and the spacings calculated from individual rings are given in the table.

Accompanying this change in lattice, a change in crystal-size was indicated by the broadening of the individual rings. The ring-breadth at half intensity was 0·16 mm. at liquid-air and increased to 0·25 mm. at room temperature. Brill(19) has derived the following expression relating crystal-size with ring-breadth in electron-diffraction patterns:

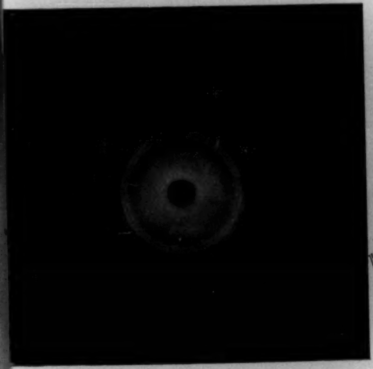
$$t = \frac{\lambda}{B' \cos \frac{1}{2}\theta},$$



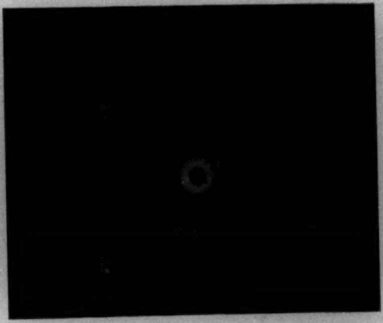
A. Indium film (20 layers) at liquid-air temperature.



B. Indium film (20 layers) at room temperature.



C. Indium film (30 layers) partially oxidized.



D. Indium film (44 layers) almost completely oxidized.

Table

Indices of ring	At liquid-air temperature		At room temperature	
	a (A.) ($c/a=1{\cdot}06$)	*a* (A.) ($c/a=1{\cdot}07$)	*a* (A.) ($c/a=1{\cdot}06$)	*a* (A.) ($c/a=1{\cdot}07$)
111	$4{\cdot}60_5$	4·59	4·61	$4{\cdot}59_5$
002	4·65	$4{\cdot}60_5$	4·63	$4{\cdot}58_5$
200	4·61	4·61	4·62	4·62
202	4·63	4·61	4·62	4·60
220	4·61	4·61	4·62	4·62
300	4·62	4·62	—	—
113	4·65	$4{\cdot}61_5$	4·62	$4{\cdot}58_5$
311	4·62	$4{\cdot}61_5$	4·62	$4{\cdot}61_5$
222	$4{\cdot}62_5$	4·61	$4{\cdot}62_5$	4·61
313 330	$4{\cdot}62_5$	$4{\cdot}60_5$	4·61	4·59
331	$4{\cdot}61_5$	4·61	—	—
402 420	$4{\cdot}61_5$	$4{\cdot}61_5$	$4{\cdot}62_5$	4·62
224	$4{\cdot}63_5$	4·61	—	—
422	4·62	$4{\cdot}61_5$	4·62	$4{\cdot}61_5$

where B' is the ideal angular breadth of a ring, λ is the wave-length, $\theta/2$ is the angle of incidence of the electron beam, and t is the mean thickness of the individual crystallites diffracting the beam. The formula is derived for substances crystallizing in the cubic system, so that the crystallites are regarded as cubes of mean edge t. As indium has an axial ratio (1·077) very near to unity, this formula can be applied to the present case without serious error. The ideal breadth B' is related to the measured ring-breadth at half intensity B by

$$B=B'+b,$$

where b is the angular width of the diffracted beam itself.

In this way it is possible to calculate the mean size of the individual particles comprising the indium film. From this calculation an interesting result emerged. Taking a film which, assuming a uniform distribution of atoms, had a thickness of 104 A. (40 layers of indium atoms) the mean particle-size immediately after deposition at liquid-air temperature was found to be 210 A. In other words, considerable aggregation must have occurred during the formation of the film, so that the electron beam encounters a field covered with isolated crystallites. At this temperature it was only possible to get good diffraction patterns through certain parts of the film; the remainder of it must have been covered either with crystallites too small (less than 10 A.) to give a pattern that could be distinguished from the diffuse scattering due to the cellulose substrate, or with non-crystalline arrangements of atoms. On raising the film to room temperature the particle-size dropped to 90 A. and this was accompanied by a great increase in the area over which a good diffraction pattern could be obtained. The rise in temperature must have caused a spreading of the larger particles or the formation of new crystallites of medium size from previously amorphous atoms or very small aggregates. The increased mobility of the atoms in the surface would facilitate this process. It should be remarked that these changes in crystal-size are irreversible: cooling from room temperature back to liquid-air temperature caused no appreciable change in the ring pattern.

Similar results were obtained from indium films of various thicknesses between 10 and 40 layers; the film-thickness had no appreciable effect upon either the lattice constants or the relative intensities of the rings. In no case was any preferred orientation of atomic planes in the film evident at any temperature.

Rather different results were obtained, however, from films formed by very slow deposition. At liquid-air temperatures these give ring patterns with the same constants as rapidly deposited films: $a = 4{\cdot}61$ A. and $c/a = 1{\cdot}07$. On raising them to room temperature the pattern changed and approximated to a face-centred cubic structure, the constants being $a = 4{\cdot}69$ A. and $c/a = 1{\cdot}02$. On admitting a small amount of air, leaving it in contact with the film for some hours and then evacuating again, the structure proved to be $a = 4{\cdot}72$ A. and $c/a = 1{\cdot}01$, and oxide rings appeared faintly. A film exposed to air for a week showed a cubic structure with $a = 4{\cdot}74$ A., and oxide rings of medium strength.

This change towards a cubic structure would appear, therefore, to be connected with oxide-formation, and it is significant that In_2O_3 has a cubic structure in which the indium atoms occupy positions corresponding to a face-centred cubic lattice with an edge of 5·06 A., as compared with the face-centred tetragonal structure, for which $a = 4{\cdot}58$ A., of the pure bulk metal. When the films are deposited rapidly this marked change is not observed, nor are the films so readily oxidized. But the slight change, from 4·61 A. and 1·07 to 4·62 A. and 1·06, in raising such films from liquid-air to room temperatures is in the same direction as that in slowly deposited films. On exposure to air the former undergo a further change to 4·67 A. and 1·03. These facts suggest that residual gas interferes with film-formation when it takes place slowly, but not appreciably with rapid formation.

When the oxidation of rapidly deposited films is allowed to proceed to an observable extent, it is found to have little or no effect on the structure of the still unoxidized metal remaining in the film. C in the Plate shows a pattern obtained in this way in which the metal and oxide rings are of similar intensity; the metal constants are here 4·63 A. and 1·06, and the oxide cube edge is 10·16 A., as compared with the value of 10·12 A. given by Wyckoff(20). In such cases, the oxide must be formed as a surface layer on the large indium crystals that are formed by rapid deposition. Continued oxidation of indium films produced sharp patterns (D in the plate) giving a slightly larger value for the oxide structure, 10·18 A.

Resistivity-measurements. In view of Lovell's results it is not possible to deduce much from the resistivity-measurements made in the present work under conditions which, though definitely under better control than those of many previous experiments, were still far from ideal owing mainly to the incomplete elimination of gas. At the same time no previous experiments have been recorded for indium, and moreover the number of atoms laid down in each film was accurately known in the experiments described.

Results have been obtained which are in several respects similar to those of earlier workers with other metals. (1) According to the conditions of deposition and thickness the resistivity of the films varies from 10^4 to 10^{-1}, whereas indium in bulk gives the value $8{\cdot}4 \times 10^{-6}$. (2) The films show a fall in resistance when raised in temperature, part of the fall being irreversible. Thus figure 2, curve 1, gives the change in resistance of a fresh film deposited at liquid-air temperature and then raised to room temperature in situ. On cooling the film again to the same temperature, curve 2 is followed. (3) There is however another effect superposed and opposite in sign. At any temperature the films age slowly with time, with a gradual rise in resistance; thus on reheating of the film, curve 3 results. In form curve 3 does not depart very seriously from curve 2, and this suggests a slow ageing of the film which otherwise changes its resistance more or less reversibly with temperature. Lovell associates a similar but more rapid change that occurs in thinner films with the existence of gas films on the substrate; when these films were reduced to a minimum, he was able to produce films stable with time. In the present conditions, the slow oxidation of the film, which the diffraction results have demonstrated, must play an important part also in increasing the resistance. (4) These films, then, can be considered as having a negative temperature coefficient. Using films of different thickness it was found that the mean temperature coefficient decreased with decreasing resistance of the film, as shown in figure 3. In this graph have been plotted the results for all films investigated, prepared under whatever conditions, in order to show the general relationship between coefficient and resistance(21); in given conditions, the resistance will be proportional to the thickness of the film. (5) The higher the working vacuum the lower was the resistivity of the film formed. When the residual pressure was 10^{-3} mm. the first measurable conductivity appeared at a thickness of 160 layers, whereas at 10^{-5} mm. a film of 30 layers already conducted. It must be remembered that at a pressure

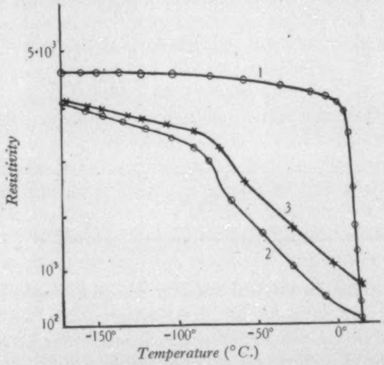


Figure 2. Change in resistivity of film (600 A. thick) with temperature.

even of 10^{-5} mm. the number of gas atoms bombarding the surface considerably exceeds the number of metal atoms which arrive there during deposition. If after deposition the pressure was changed from 10^{-5} mm. to atmospheric, the resistance rose by as much as tenfold in some cases. This was probably mostly due to oxidation, because the effect was greater in oxygen than in air. Moreover evidence of oxide-formation has been given above in the determinations of structure. But the fact that there is still some residual effect in nitrogen suggests that sorption of additional gas does also play some part in the phenomenon. (6) When the indium was deposited in turn on substrates of varying expansibilities, no significant changes were observed in the value of the temperature coefficient. The substrates used and their coefficients of expansion were: glass ($8{\cdot}5 \times 10^{-6}$), bakelite ($2{\cdot}8 \times 10^{-5}$), ice ($2{\cdot}7 \times 10^{-5}$), cellulose ($1{\cdot}4 \times 10^{-4}$) and paraffin ($1{\cdot}1 \times 10^{-4}$), as compared with that of indium in bulk ($2{\cdot}5 \times 10^{-5}$). It appeared, therefore, that the difference in expansibility between film and substrate had no appreciable effect on the change in resistance of the film with temperature.

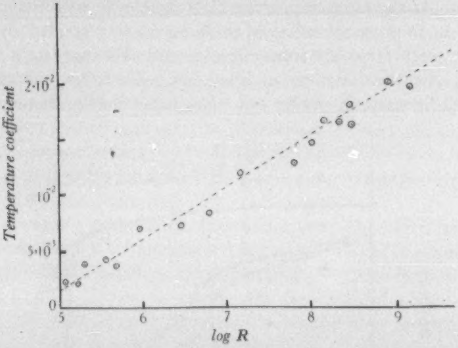


Figure 3. Change in temperature coefficient of films with resistance, independent of conditions of deposition.

§ 4. DISCUSSION

The behaviour of thin metallic films with temperature change has given rise to much discussion in the literature on their resistance. The negative temperature coefficient appears to be a property of such films prepared in the presence of appreciable residual gas, as is also their high resistivity. These characteristics are evidence that the films cannot be continuous. They differ, therefore, from the films which Lovell obtained that are quite stable, although only a few atomic layers in thickness, having the positive coefficient to be expected of a coherent uniform metallic surface.

At the same time, the structure determinations show that the crystallites which compose the film have a lattice which is only slightly different from that of the bulk metal. We must conclude therefore that a freshly deposited film is made up of a number of isolated crystalline particles, and that this island formation is responsible for the high resistivity. With rise in temperature it has been shown that a decrease in mean particle-size takes place, not at the expense of the large crystallites first formed, but by the formation of particles of medium size from those which were previously too small to give a sharp diffraction pattern. A similar process of aggregation has been observed by Andrade(22) in gold and silver films at a much higher temperature. It seems reasonable to associate this recrystallization with the irreversible part of the resistance decrease which occurs when a film is raised initially from liquid-air to room temperature.

With regard to the negative temperature coefficient, it is difficult to explain it as some have attempted to do(3, 10) by simple and regular making and breaking of bridges between crystalline island formations. Such a process could not occur to any large extent unless assisted by differing expansibilities of film and substrate. Bartlett(10), indeed, suggested that such a process gave rise to the negative temperature effect. But the present work has shown that substrates of very different expansibilities produce no appreciable variation in the coefficient. The making and breaking of contacts between different parts of the film that may take place in this way must therefore make little difference to the total resistance.

The influence of residual gas upon the film resistance has been demonstrated and suggests that this may be responsible also for the anomalous temperature coefficient. In particular, the fact that nitrogen as well as oxygen affects the film points to sorption as well as oxidation occurring. It has been shown(6) that sputtered films may contain occluded gas equal in volume to the metal itself. In the present work, the vacuum was much higher, but the deposition temperature much lower than in sputtering experiments. It is possible, therefore, that gas is absorbed in the film during formation and consequently increases the resistance, even if direct oxidation is negligible. As the temperature rises, desorption of the gas would result in a fall in the resistance. On again decreasing the temperature, resorption would occur and give rise to the reversible effect observed.

This explanation receives support from the results of the present electron-diffraction work and from a similar investigation recently made by Riedmiller(18) with films of nickel, silver and gold. He found that evaporated nickel films had a density lower by 18 per cent than that of the bulk metal and a lattice constant 1·6 per cent greater. This increase he ascribed to absorption of gas in the lattice itself; the major part of the density-change he supposed to be due to sorption of gas in a loose agglomerate of crystallites. These films were deposited at room temperature; at the very much lower deposition temperature of the present work more extensive absorption is to be expected. The determination of the structure of freshly deposited indium films (above) showed that the lattice constant is about 0·7 per cent greater than that of the bulk metal. There must, therefore, be less gas in the lattice itself than in Riedmiller's experiments, which may be bound up with the fact that the indium crystallites were ten times larger than his nickel ones. He found also that the gold films showed a 0·8-per-cent and the silver a 1-per-cent increase in size over the normal lattice. Exposure of the nickel films to air increased the discrepancy from 1·2 to 1·4 per cent, and to hydrogen up to 1·8 per cent; the nickel oxide formed had a lattice constant 2·4 per cent greater than normal. No observations of change in lattice with temperature are reported.

In the present work, the increases in spacing are of the same order; 0·7 per cent for indium and 0·6 per cent for indium oxide in rapidly deposited films, which show little variation with temperature. Films that are slowly deposited show a rapid increase in size of lattice with temperature-rise: at room temperature it is 2·4 per cent greater than normal and this increases to 3·1 per cent on short exposure to air, faint oxide rings appearing. Rapidly deposited films show a total increase of 2·0 per cent after exposure to air. In both cases the axial ratio changes also towards a more nearly cubic structure; the longest oxidation allowed produced a film of cubic structure, within the 1-per-cent error of estimation, with an indium lattice 3·4 per cent greater than normal. The drift towards the cubic structure of indium oxide, and the ready appearance of oxide rings, leave no doubt that these changes in lattice constant arise from interference of gas in the crystal lattice, as distinct from sorption into a porous film. It may be that sorption of gas or oxide-formation in the surface layers of small crystallites causes distortion of the inner metal lattice. On the other hand, it may be that intermediate stages in the formation of the oxide are being observed, oxygen first entering the lattice and modifying the metallic structure before actual formation of indium oxide crystals takes place. The largest indium spacing observed in a partially oxidized film was 4·74 A., whereas the separation of the indium atoms in the normal oxide is 5·06 A.

Whatever the process occurring, it has been established that the metal lattice becomes more nearly cubic as oxidation proceeds. A small change in this direction was observed on raising rapidly deposited films to room temperature, even when the vacuum was maintained at 10^{-5} mm. and no sign of oxide rings appeared. Hence residual gas is affecting the film structure, even at this low pressure, and this process must play a part in the slow ageing of the films—the drift towards high resistance that continues even in high vacua. In order to investigate adequately the suggested influence of gas in this way and on the negative temperature coefficient, it would be necessary to work with a very much higher vacuum than that obtainable in an electron-diffraction apparatus of the type here employed. In such conditions it would be possible to obtain films of normal resistivity and temperature coefficient, as Lovell has done with rubidium and caesium, and to investigate their structure.

5. ACKNOWLEDGEMENTS

The author wishes to express his indebtedness to Prof. A. M. Tyndall for affording the facilities that made this work possible and for his help and interest in it; and to Dr E. T. S. Appleyard for valuable advice and discussion.

REFERENCES

(1) STONE, *Phys. Rev.* **6**, 1 (1898); OBERBECK, *Ann. Phys.*, Lpz., **47**, 353 (1892); and others.

(2) LONGDEN, *Phys. Rev.* **9**, 40 (1900); PATTERSON, *Phil. Mag.* (6), **4**, 652 (1902).

(3) REINDERS and HAMBURGER, *Ann. Phys.*, Lpz., **10**, 649, 790, 905 (1931); **11**, 40 (1931); SUHRMANN and BARTH, *Phys. Z.* **36**, 843 (1935). Summaries of earlier work: RIEDE, *Z. Phys.* **28**, 177 (1924); BARTLETT, *Phil. Mag.* (7), **5**, 848 (1928).

(4) LOVELL, *Nature*, Lond., **137**, 493 (1936); *Proc. roy. Soc.* (in the Press).

(5) COCKCROFT, *Proc. Roy. Soc.* A, **119**, 293 (1928).

(6) BÜSSEM and GROSS, *Z. Phys.* **86**, 135 (1933); **87**, 778 (1934).

(7) INGERSOLL and DEVINNEY, *Phys. Rev.* **26**, 86 (1925); KAHLER, *Phys. Rev.* **18**, 210 (1921); and KRAMERS, ref. (9) (below).

(8) STEINBERG, *Phys. Rev.* **21**, 22 (1923); DEMBINSKA, *Z. Phys.* **54**, 46 (1929).

(9) FÉRY, J. *Phys. Radium*, **9**, 38 (1928); KRAMERS, *Ann. Phys.*, Lpz., **19**, 37 (1934); and PERUCCA, *Ann. Phys.*, Lpz., **4**, 252 (1930); *Z. Phys.* **91**, 660 (1934).

(10) Cf. BARTLETT (ref. 3), REINDERS and HAMBURGER (ref. 3), and SWANN, *Phil. Mag.* (6), **28**, 467 (1914).

(11) KIRCHNER, *Z. Phys.* **76**, 576 (1932).

(12) PRINS, *Z. Krystallogr.* **86**, 301 (1933); *Nature*, Lond., **131**, 760 (1933).

(13) POWELL and BRATA, *Proc. roy. Soc.* A, **141**, 463 (1933).

(14) GOLDSCHMIDT, *Ann. Phys.*, Lpz., **82**, 947 (1927).

(15) COSSLETT, *Trans. Faraday Soc.* **30**, 981 (1934).

(16) HULL and DAVEY, *Phys. Rev.* **17**, 549, 571 (1921).

(17) WYCKOFF, *Structure of Crystals* (Supplement 1930–4), p. 10.

(18) RIEDMILLER, *Z. Phys.* **102**, 408 (1936).

(19) BRILL, *Kolloidzschr.* **69**, 301 (1934); *Z. Krystallogr.* **87**, 275 (1934).

(20) WYCKOFF, *Structure of Crystals* (2nd ed.), p. 254.

(21) Cf. DEAGLIO, *Z. Phys.* **91**, 657 (1934).

(22) ANDRADE, *Trans. Faraday Soc.* **31**, 1137 (1935); ANDRADE and MARTINDALE, *Philos. Trans.* **235**, 69 (1935).

DISCUSSION

Dr L. HARTSHORN. I am particularly interested in the mechanism of electrical conductivity in these thin films. It seems clear from the author's experiments that the low conductivity and abnormal temperature coefficients are associated in some way with discontinuities in the film and the inclusion of gas molecules. It is however by no means obvious why the adsorption of gas molecules should cause a decrease of conductance, and why subsequent desorption should cause an increase. Can the author give us any idea of the mechanism of these changes? Is the electrical conductivity of an adsorbed layer of gas molecules greater or less than that of the space left when it is desorbed? Again, why should an increase of particle-size be associated with an increase of conductance? Is the change of particle-size limited to two dimensions?

I am puzzled by the statement that sputtered films may contain up to their own volume of gas. How is this volume of gas supposed to be measured? Does it mean that the number of gas molecules is of the same order as the number of atoms of the metal?

It is very curious that the theory of island-formation, the first and most obvious of the explanations advanced to explain the low electrical conductance of thin metal films, should be substantiated after the publication of so much apparent evidence to the contrary. The resistance of a discontinuous film should be different for alternating and direct currents, and we should certainly not expect to find a low electrical conductance at very high frequencies if the constituent particles possess normal properties. Again it has been claimed that the abnormal resistance vanishes at a temperature characteristic of the metal and not of the method of preparation of the film. Must we dismiss the evidence of this character as inadequate, or is it possible that the films of platinum, antimony, silver, etc. used by previous workers behave quite differently from those of indium, which does not appear to have been used previously for such work?

Mr L. R. G. TRELOAR. In connexion with the measurement of the beam-intensity by the ionization of the indium atoms by hot tungsten oxide, I should like to know whether the author is aware of any published data on the work function of the latter material. It also occurred to me that, at the filament-temperatures indicated in the paper, there might be some tendency for an appreciable proportion of the indium atoms to remain adsorbed on the oxide surface. Such an effect, if it occurred, might very well reduce the work function to an extent sufficient to cause the proportion of indium atoms ionized to fall below 100 per cent, thus giving an incorrect value of beam-intensity. It would be interesting to know whether the author has had any difficulties of this kind.

Mr D. A. WRIGHT. Although the author has been concerned primarily with the behaviour of films deposited on a cellulose substrate, I note that some study has been made of other substrates. I am particularly interested in the behaviour of thin metallic films on glass, and should like to know of the author's experience with this material, with respect to both the structure and the conductivity of the films. Has he observed crystal aggregates, and if so are their dimensions under corresponding conditions similar to those of the aggregates on cellulose? I note that the temperature coefficient of conductivity has been found to be independent of the substrate. Is this true of the actual value of the conductivity, and more particularly of the details of the irreversible ageing process? If any modification has been noticed in the case of glass, has any study been made of the variations produced by different methods of preparing the glass surfaces, e.g. by different methods of washing or by different heat treatments? I think the state of the surface will be very important, since it will certainly modify the sorption of gas upon it. It may also influence the possible production of gas from the body of the substrate, while the possibility of chemical action between the substrate and metal must not be overlooked. The state of the surface must I think be added to the other factors discussed, in seeking to explain the large variations among the results obtained by different workers.

AUTHOR's reply. Referring to Dr Hartshorn's remarks on the mechanism of conductivity in these thin films, I must make it clear that I do not advance any single, simple explanation of this mechanism; island-formation alone cannot account for the properties of the films. The absorption of gas and the influence of surface conditions (as shown by Lovell) must enter into any complete explanation. So that these disturbing factors would not allow of a simple deduction from a.-c. measurements of resistance, although it would be interesting to have such results. The irreversible increase in particle size must be associated with improved means of communication across the film, especially if it occurs at the expense of small semi-crystalline or colloidal aggregates, as Kirchner's work suggests. In any case, it is a fact that increase in temperature immediately after film-formation results in a large irreversible increase in conductivity and in crystal size; it seems probable, *a priori*, that this latter change would be mainly two-dimensional.

The transition temperature, reported by Kramers, was not observed in this work or by Lovell, and the recent work of Suhrmann and Barth with other metals at lower temperatures and under better vacuum conditions failed to find any such effect; it seems probable that it was due to some adventitious factor in Kramers's experimental conditions. In reference to the results of Büssem and Gross, the statement that "sputtered films may contain up to their own volume of gas" means that the number of gas atoms in the films may be equal to the number of metal atoms.

With reference to Mr Treloar's remarks: the collector system was found to function perfectly and there was no evidence of accumulation of indium atoms on the surface at rates of deposition up to ten atomic layers per minute. The system has been in use in this laboratory for some time, and is fully described by Powell and Mercer, who give the electronic work function of the tungsten oxide surface as 6·13 volts, compared with an ionization potential of 5·76 volts for indium.

All the electron-diffraction results in this work were obtained from indium films deposited on collodion films, so that I am unable to give Mr Wright any information as to the structure of films deposited on glass. It was not possible to obtain other substrates sufficiently thin to transmit electrons at the voltage used, and no attempt was made to get reflection patterns from metal films on thick substrates. So far as the conductivity is concerned, no significant difference was found between films deposited on collodion, glass, or other substrates; this would suggest that there are no important structural differences on substrates not carefully cleaned. On specially cleaned surfaces other results might be expected. As reported, Lovell has shown that extraordinarily careful cleaning of a glass substrate is necessary in order to get films of nearly normal conductivity; it would be very interesting to investigate the structure of such films, by reflection methods in a high vacuum apparatus. I agree that the surface state may be all important in determining the structure of the deposited film.

THE EFFECT OF ORIENTATION ON STRESSES IN SINGLE CRYSTALS AND OF RANDOM ORIENTATION ON STRENGTH OF POLYCRYSTALLINE AGGREGATES

By H. L. COX, B.A., and
D. G. SOPWITH, B.Sc.Tech., A.M.I.Mech.E.,
The Engineering Department, National Physical Laboratory

Received 21 *October* 1936. *Read in title* 11 *December* 1936

ABSTRACT. A method of tabulating resolved shear stresses for any type of slip system has been devised. Diagrams of maximum resolved shear stress for torsion and tension as functions of orientation have been drawn both for the case of aluminium, with slip plane of type (111) and slip direction of type (110), and for the case of iron with slip direction of type (111) and any slip plane. Similar diagrams of harmonic mean stress in torsion also have been drawn. The effect of different criteria of failure in indicating the failure of polycrystalline specimens has been investigated. If elastic failure of polycrystalline specimens is determined entirely by the maximum resolved shear stress developed locally, the ratio of the yield in torsion to the yield in tension should be 0·500; whereas if elastic failure is determined by the complete yield of all the crystals of the specimen, the value of this ratio should be about 0·577.

§ 1. INTRODUCTION

The results of numerous experiments on single crystals of several different metals have shown that plastic deformation under stress often occurs entirely by slip along certain crystallographic planes in certain crystallographic directions(1, 2). For instance, in aluminium and other metals conforming to the face-centred cubic lattice, the possible slip planes are the four octahedral planes, and in each slip plane the intersections with the other three octahedral planes are possible slip directions. Any one of the possible combinations of slip plane and slip direction will for convenience be termed a *slip-plane-direction*.

Now, in the majority of metals, all the slip-plane-directions are crystallographically identical, so that if slip under stress occurs upon any one of them in preference to the others, the reason for this preference must be sought in the stress conditions imposed. By measurement of the deformation of single crystals of aluminium produced by static tensile stresses, Prof. G. I. Taylor(2) was able to show that in general slip occurred entirely in one slip-plane-direction and that the particular slip-plane-direction concerned was that for which the shear stress resolved on the slip plane and along the slip direction was a maximum. The shear stress thus associated with each slip-plane-direction will for convenience be termed a *resolved shear stress*. Other experiments on single crystals of aluminium (3) and other metals (4) under many different types (5, 6) of stress system have shown, practically without exception, that deformation always occurs by slip in the slip-plane-direction subjected to the greatest resolved shear stress. Moreover, recent tests under fatigue stresses on specimens of aluminium consisting of more than one crystal (7) have indicated both that the effect of the intercrystalline boundaries on the stress-distribution is extremely slight and that the deformation of the individual crystals is again determined by the values of the resolved shear stresses. The determination of the values of the resolved shear stresses is therefore an essential part of any investigation of the strength of metals in relation to their crystalline structure. It is the purpose of this paper to discuss the methods by which these stresses may be determined and to state certain general conclusions that may be drawn.

§ 2. DETERMINATION OF THE PRINCIPAL STRESSES

The distribution of stress set up in any specimen by the application to its boundaries of known forces and displacements may be calculated by the theory of elasticity, by means of the relations between stress and strain appropriate to the elastic symmetry of the material of which the specimen is composed, but if the boundary conditions imposed relate to stresses alone the constant or linear stress-distribution set up is independent of the type of elastic symmetry (8). The stress-distribution set up in a circularly (or elliptically) cylindrical specimen of aeolotropic material by the application of equal and opposite torsional couples is therefore the same as the stress-distribution that would be set up by the same couples applied to a similar cylinder of isotropic material (8). The same conclusion applies also to bending by pure couples and to combined bending and torsion. The stress-distribution set up in single-crystal specimens of circularly cylindrical form by tension, torsion, flexure or any combination of these types of stress, may therefore be regarded as known, and the determination of the values of the resolved shear stresses reduces merely to the resolution of the known principal stresses along the slip-plane-directions.

§ 3. DETERMINATION OF RESOLVED SHEAR STRESSES

In order to facilitate the presentation of the theoretical analysis, attention will be confined mainly to the face-centred cubic lattice. Many of the results are, however, applicable with little or no alteration to other lattices; the necessary alterations in the case of the body-centred lattice will be indicated. In the face-centred cubic lattice, the possible slip planes are the four octahedral planes of which the Millerian indices are (111), $(\bar{1}11)$, $(1\bar{1}1)$ and $(\bar{1}\bar{1}1)$; *these planes will be termed* 0, 1, 2 *and* 3 *respectively*:* the possible slip directions are the six intersections of the slip planes in pairs (the normals to the planes of the type 110); these directions will be termed 01, 02 etc.,

---

* Note that the numeral corresponds with the position of the bar above the Millerian indices.

01 being the intersection of the planes 0 and 1, i.e. the normal to the plane $01\bar{1}$. Since all the slip planes are indistinguishable, the numbers 0, 1, 2, 3 (and the corresponding Millerian indices) can be assigned in 24 ways; in order to avoid this ambiguity, the axis of the specimen will be taken as lying within the solid angle bounded by the three planes through the normal to the cube face 001, the normal to the slip plane 0 (111) and the slip direction 13 (101) in pairs. This convention is shown in the stereographic diagram, figure 1. The position of the axis of the specimen within the triangle shaded in figure 1 defines two of the three parameters necessary to determine completely the orientation of the crystal with respect to the principal axes of stress. The third parameter may be represented by a rotation about the specimen-axis; but for tension, torsion, and simple flexure such a rotation does not alter the stress-distribution,* so that the position of the axis defines the resolved shear stresses completely. For combined bending and torsion all three parameters must be specified.

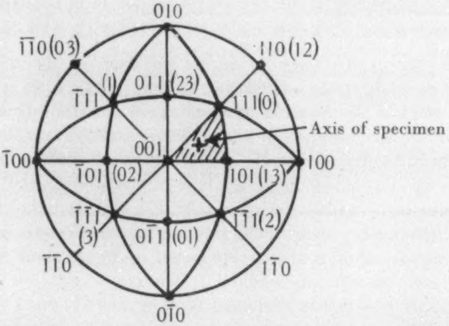


Figure 1

The orientations of the various crystallographic directions are most conveniently specified by spherical co-ordinates referred to the axis of the specimen and to a reference plane, arbitrarily chosen, through that axis; the spherical co-ordinates of the normal to the plane 0 will be written (θ_0, ψ_0), of the direction 01 (θ_{01}, ψ_{01}), and so on. The position of any point on the surface of the specimen will be specified by the angle λ between the diametral plane through the point and the reference plane. General tensile stresses will be denoted by p, general shear stresses by s; the maximum tensile stress parallel to the axis of the specimen will be denoted by P, the actual maximum shear stress due to a torque T by S (which is equal to $2T/\pi r^3$ where r is the radius of the specimen). The resolved shear stress in the slip-plane-direction 01 will be denoted by S_{01}, etc. (N.B. S_{01} is *not* in general equal to S_{10}.)

In the case of simple tension

$$S_{01} = P \cos\theta_0 \cos\theta_{01}, \text{ etc.} \qquad \text{......(1).}$$

---

* In the case of flexure, a rotation about the specimen-axis alters the position of the stress system but not its magnitude.

In the case of simple bending $S_{01}=P\cos\lambda\cos\theta_0\cos\theta_{01}$, the plane of bending being taken as reference plane; but the factor $\cos\lambda$ affects all the resolved shear stresses equally, so that the case of simple bending is effectively the same as that of simple tension. The greatest resolved shear stress is defined by the greatest of the products $\cos\theta_0\cos\theta_{01}$, etc. With the convention adopted in figure 1 this is $\cos\theta_2\cos\theta_{23}$; the manner in which the ratio S_{23}/P varies with the orientation will be discussed later.

In the case of simple torsion S_{01}, etc. can be expressed in the form

$$S_{01}=SA\cos(\lambda-\alpha) \qquad \ldots\ldots(2)^{(5)},$$

where
$$A=\sqrt{\{-\tfrac{1}{2}(\cos 2\theta_0+\cos 2\theta_{01}+2\cos 2\theta_0\cos 2\theta_{01})\}}$$

and
$$\alpha-\psi_0=\tan^{-1}[(\tan^2\theta-1)\cot(\psi_{01}-\psi_0)].*$$

Calculation of the values of the ratios S_{01}/S, etc. by determination of the values of A and α from the equations (2) is a fairly simple process; but the values have to be determined afresh for each different orientation, and if many crystals are concerned the work becomes laborious. If the ratios S_{01}/S can be determined as functions of the position of the axis (within the triangle shaded in figure 1), the necessity for the intermediate calculation of the values of $(\theta_0\psi_0)$, $(\theta_{01}\psi_{01})$, etc. is avoided and moreover it becomes feasible to tabulate values of S_{01}/S, etc. for all possible orientations.

Let (θ, ϕ) be the spherical co-ordinates of the axis of the specimen referred to the normal to the cube face 001 as axis and the plane 010 as reference plane. As reference plane associated with the axis of the specimen as axis take the plane through the normal to the cube face 001. Let the axes $(Oxyz)$ be Ox perpendicular to the plane 100, Oy perpendicular to 010, and Oz perpendicular to 001; and let the axes $(Ox'y'z')$ be Oy' perpendicular to the reference plane through the axis of the specimen and the axis Oz, Oz' the axis of the specimen, and Ox' the perpendicular to Oy' and Oz'. Then transformation from the axes $(Oxyz)$ to the axes $(Ox'y'z')$ is effected by means of the table

$$\left.\begin{array}{cccc} & x' & y' & z' \\ x & \cos\theta\cos\phi & -\sin\phi & \sin\theta\cos\phi \\ y & \cos\theta\sin\phi & \cos\phi & \sin\theta\sin\phi \\ z & -\sin\theta & 0 & \cos\theta \end{array}\right\} \qquad \ldots\ldots(3).$$

Now, if the surface of the specimen is free from traction, at any point on the surface, one principal stress is normal to this surface and is zero. The stress system may therefore be specified by the values of the principal stresses p_1 and p_2 (p_1 being greater than p_2) and the angle β which the direction of p_1 makes with the axis of the specimen. If λ is the angle which the plane through the point considered and the axis of the specimen makes with the reference plane $y'=0$, the direction cosines with respect to $(Ox'y'z')$ of the directions of the stresses p_1 and p_2 are:

$$p_1(-\sin\beta\sin\lambda,\ \sin\beta\cos\lambda,\ \cos\beta),$$
$$p_2(\cos\beta\sin\lambda,\ -\cos\beta\cos\lambda,\ \sin\beta).$$

---

* Change of the reference plane has the effect of adding a constant angle to λ and to the values of ψ. α is therefore increased by the same constant angle, so that $\lambda-\alpha$ remains invariant.

From the table, (3), the direction cosines $p_1\,(l_1 m_1 n_1)$ and $p_2\,(l_2 m_2 n_2)$ with respect to the axes $(Oxyz)$ are found to be

$$l_1 = -\sin\beta\sin\lambda\cos\theta\cos\phi - \sin\beta\cos\lambda\sin\phi + \cos\beta\sin\theta\cos\phi,$$
$$m_1 = -\sin\beta\sin\lambda\cos\theta\sin\phi + \sin\beta\cos\lambda\cos\phi + \cos\beta\sin\theta\sin\phi,$$
$$n_1 = \sin\beta\sin\lambda\sin\theta + \cos\beta\cos\theta.$$

Similar relations hold for (l_2, m_2, n_2).

By equation (1), the resolved shear stress

$$S_{01} = p_1\cos\theta'_0\cos\theta'_{01} + p_2\cos\theta''_0\cos\theta''_{01},$$

where θ_0' is the angle between the normal to the plane 0 and the direction of the stress p_1, etc.; but the direction cosines of the normal to the plane 0 referred to the axes $(Oxyz)$ are $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$ and of the direction 01 $(0, 1/\sqrt{2}, -1/\sqrt{2})$, therefore:

$$\cos\theta'_0 = (l_1 + m_1 + n_1)/\sqrt{3}$$

and

$$\cos\theta'_{01} = (m_1 - n_1)/\sqrt{2},$$

with similar formulae for $\cos\theta''_0$, $\cos\theta'_1$, etc. etc.* Therefore

$$\sqrt{6}\,.\,S_{01} = p_1\,(l_1+m_1+n_1)\,(m_1-n_1) + p_2\,(l_2+m_2+n_2)\,(m_2-n_2).$$

Now, (l_1, m_1, n_1) (l_2, m_2, n_2) are of the form

$$l_1 = \sin\beta\,(A\sin\lambda + B\cos\lambda) + C\cos\beta, \quad l_2 = -\cos\beta\,(A\sin\lambda + B\cos\lambda) + C\sin\beta;$$
$$m_1 = \sin\beta\,(P\sin\lambda + Q\cos\lambda) + R\cos\beta, \quad m_2 = -\cos\beta\,(P\sin\lambda + Q\cos\lambda) + R\sin\beta;$$
$$n_1 = \sin\beta\,.\,L\sin\lambda + M\cos\beta, \quad n_2 = -\cos\beta\,.\,L\sin\lambda + M\sin\beta.$$

Any expression of the form

$$p_1\,(l_1+m_1+n_1)\,(m_1-n_1) + p_2\,(l_2+m_2+n_2)\,(m_2-n_2)\dagger$$

may therefore be written in the form:

$$(p_1\sin^2\beta + p_2\cos^2\beta)\,\{(A+P+L)\sin\lambda + (B+Q)\cos\lambda\}\,\{(P-L)\sin\lambda + Q\cos\lambda\} \quad \ldots\ldots(4a),$$
$$+(p_1-p_2)\sin\beta\cos\beta\,[\{(A+P+L)\sin\lambda + (B+Q)\cos\lambda\}\,(R-M) + \{(P-L)\sin\lambda + Q\cos\lambda\}\,(C+R+M)] \quad \ldots\ldots(4b),$$
$$+(p_1\cos^2\beta + p_2\sin^2\beta)\,(C+R+M)\,(R-M) \quad \ldots\ldots(4c).$$

Now $p_1\sin^2\beta + p_2\cos^2\beta$ is the circumferential tension, Q,
$(p_1 - p_2)\sin\beta\cos\beta$ is the shear due to torsion, S,
and $p_1\cos^2\beta + p_2\sin^2\beta$ is the axial tension, P.

Uniform circumferential tension (produced, say, by internal pressure) gives a distribution of resolved shear stress that is a sinusoidal function of 2λ, torsion gives a distribution that is a sinusoidal function of λ, the mean value of any resolved shear stress being in this case zero. If P is due to simple tension, the resolved shear stress is independent of λ; but if P is itself multiplied by $\cos(\lambda-\gamma)$ the distribution is again simple-harmonic. It should be noticed that the value of γ, the angle between the plane of bending and the reference plane $y'=0$, affects the stress-distribution in combined bending and torsion but not in simple bending.

---

* The alterations necessary for any other type of crystal symmetry are obvious.

† In any lattice system the expression for resolved shear stress will be of this form, viz. $p_1 \times$ (homogeneous function of the 2nd degree in $l_1 m_1 n_1$) $+ p_2 \times$ (homogeneous function of the 2nd degree in $l_2 m_2 n_2$).

The expression in square brackets in (4*b*) above is the ratio S_{01}/S. Similar expressions for the other ratios may be obtained. Inserting the values of A, B, C, etc. the values of S_{01}, etc. may be expressed in the form

$$S_{01}/S=\frac{1}{\sqrt{6}}\{(E\cos 2\theta+F\sin 2\theta)\sin\lambda+(G\cos\theta+H\sin\theta)\cos\lambda\}\ \ldots\ldots(5),$$

where E, F, G and H are functions of ϕ only. The expressions for E, F, G and H are given in table 1 below.

Table 1. General stress coefficients for torsion

	E	F	G	H
S_{01}	$-\cos\phi$	$\frac{1}{2}(-\cos 2\phi+\sin 2\phi+3)$	$-\sin\phi$	$-\cos 2\phi-\sin 2\phi$
S_{02}	$-\sin\phi$	$\frac{1}{2}(\cos 2\phi+\sin 2\phi+3)$	$\cos\phi$	$-\cos 2\phi+\sin 2\phi$
S_{03}	$\cos\phi-\sin\phi$	$\cos 2\phi$	$\cos\phi+\sin\phi$	$2\sin 2\phi$
S_{12}	$-\cos\phi-\sin\phi$	$\cos 2\phi$	$\cos\phi-\sin\phi$	$2\sin 2\phi$
S_{13}	$-\sin\phi$	$\frac{1}{2}(\cos 2\phi-\sin 2\phi+3)$	$\cos\phi$	$\cos 2\phi+\sin 2\phi$
S_{10}	$\cos\phi$	$\frac{1}{2}(-\cos 2\phi-\sin 2\phi+3)$	$\sin\phi$	$\cos 2\phi-\sin 2\phi$
S_{23}	$-\cos\phi$	$\frac{1}{2}(-\cos 2\phi-\sin 2\phi+3)$	$-\sin\phi$	$\cos 2\phi-\sin 2\phi$
S_{20}	$\sin\phi$	$\frac{1}{2}(\cos 2\phi-\sin 2\phi+3)$	$-\cos\phi$	$\cos 2\phi+\sin 2\phi$
S_{21}	$-\cos\phi-\sin\phi$	$-\cos 2\phi$	$\cos\phi-\sin\phi$	$-2\sin 2\phi$
S_{30}	$\cos\phi-\sin\phi$	$-\cos 2\phi$	$\cos\phi+\sin\phi$	$-2\sin 2\phi$
S_{31}	$\sin\phi$	$\frac{1}{2}(\cos 2\phi+\sin 2\phi+3)$	$-\cos\phi$	$-\cos 2\phi+\sin 2\phi$
S_{32}	$\cos\phi$	$\frac{1}{2}(-\cos 2\phi+\sin 2\phi+3)$	$\sin\phi$	$-\cos 2\phi-\sin 2\phi$

Equation (5) is of course identical with equation (2); $E\cos 2\theta+F\sin 2\theta$ corresponds to $A\sin\alpha$ and $G\cos\theta+H\sin\theta$ to $A\cos\alpha$, the angle α being measured from the plane through the axis of the specimen and the normal to the cube face 001.* But A and α are now functions of two variables only.

§ 4. ACTUAL MAXIMUM RESOLVED SHEAR STRESSES DUE TO TENSION AND TORSION

With the convention adopted (figure 1), the greatest resolved shear stress in the face-centred lattice due to tension is always the stress S_{23}.† It is proposed to prove that the greatest resolved shear stress in this lattice due to torsion may be either S_{01}, S_{03} or S_{31} and to examine how the value of the actual maximum resolved shear stress due to tension or torsion varies with orientation.

For this purpose it is convenient to use a method of projection different from the stereographic method normally employed. If a line representing any crystallographic direction is drawn from the centre of a sphere, of which the upward-drawn vertical diameter represents the axis of reference (e.g. the positive direction of the specimen-axis), the colatitude of the point in which this line meets the sphere is the spherical co-ordinate θ and its longitude (with reference to the arbitrary reference plane) is ψ. To obtain the stereographic diagram, these points on the surface of the sphere are projected on to the equatorial plane, where $\theta = \pi/2$, from the lowest point of the sphere, where $\theta = \pi$. All directions for which $\theta < \pi/2$ are then represented by points on a plane within a circle of radius equal to the radius of the sphere; the polar co-ordinates (r, α) in this diagram of any direction (θ, ψ) are given by $r = \tan \frac{1}{2}\theta$ and $\alpha = \psi$. In the stereographic diagram, figure 1, the axis of reference used is the normal to the cube face nearest to the axis of the specimen, the axis of the specimen being shown as a point within the triangle bounded by the three planes through the normals to the planes 001, 101, 111 in pairs. In the symmetrical stereographic diagram, this elementary triangle is bounded by two straight lines and an arc of a circle; but for the present purpose it is very much more convenient to use a method of projection that makes all three sides straight. This is achieved by projecting not from the south pole but from the centre of the sphere on to the tangential plane at the north pole $(\theta = 0)$. The polar co-ordinates (r', α') of a point in this diagram representing the direction (θ, ψ) are given by $r' = \tan \theta$, $\alpha' = \psi$. Points for which $\theta \to \pi/2$ are thus projected into points that lie at a great distance from the pole of the diagram; but if attention is confined to the triangle 100, 101, 111, this disadvantage does not make itself apparent. It should be noted that the diagram obtained by this central projection consists merely in the intersections of the directions, etc. with the tangent plane to the sphere at the north pole. Planes project therefore into straight lines, whereas in the stereographic projection all planes, except those through the axis, project into circular arcs.

---

* In determining the effect of combined bending and torsion, it is necessary to ascertain the sense of the shear stresses due to torsion. If the normals to the slip planes marked 0, 1, 2, 3 and the directions marked 01, 02, etc. (note particularly 03 and 12) in figure 1 are taken as positive, the values of E, F, G, H tabulated give the resolved shear stresses due to a right-handed torque. Note that α and λ are measured in the opposite sense, viz. anti-clockwise in figure 1.

† This may be proved very simply by comparison of the expressions of the type (4*c*).

The position of the axis within the elementary triangle may be specified by its direction cosines a, b and c $(c > a > b)$; or, if in the central projection, the normal to the plane 001 is taken as origin and the lines $\psi = 0$ and $\psi = \pi/2$ as axes of x and y, the position of the axis may be defined by $x = a/c$ and $y = b/c$. The values of θ_0 and θ_{01} are then given by:

$$\left.\begin{aligned} \cos\theta_0 &= \frac{1}{\sqrt{3}}(a+b+c) = \frac{1+x+y}{\sqrt{\{3(1+x^2+y^2)\}}} \\ \cos\theta_{01} &= \frac{1}{\sqrt{2}}(b-c) = \frac{y-1}{\sqrt{\{2(1+x^2+y^2)\}}} \end{aligned}\right\} \qquad \ldots\ldots(6).$$

Similar expressions for all the other θ values may be written down. From equations (2) above

$$\begin{aligned} A^2_{01} &= -\tfrac{1}{2}(\cos 2\theta_0 + \cos 2\theta_{01} + 2\cos 2\theta_0 \cos 2\theta_{01}) \\ &= (\cos 2\theta_0 + \cos^2\theta_{01} - 4\cos^2\theta_0 \cos^2\theta_{01}). \end{aligned}$$

Using the relations of the type (6), we find

$$\left.\begin{aligned} 6A^2_{01} &= 2(a+b+c)^2 + 3(b-c)^2 - 4(b-c)^2(a+b+c)^2 \\ 6A^2_{02} &= 2(a+b+c)^2 + 3(c-a)^2 - 4(c-a)^2(a+b+c)^2 \end{aligned}\right\} \qquad \ldots\ldots(7),$$

and similar expressions for $6A^2_{03}$, etc.

Taking differences and remembering that $c > a > b > 0$ and that $a^2+b^2+c^2=1$, we have

$$6\,(A^2_{01}-A^2_{02}) = \{(b-c)^2-(c-a)^2\}\,\{3-4\,(a+b+c)^2\}$$
$$= (b-a)\,(a+b-2c)\,(-1-8\Sigma bc) < 0.$$

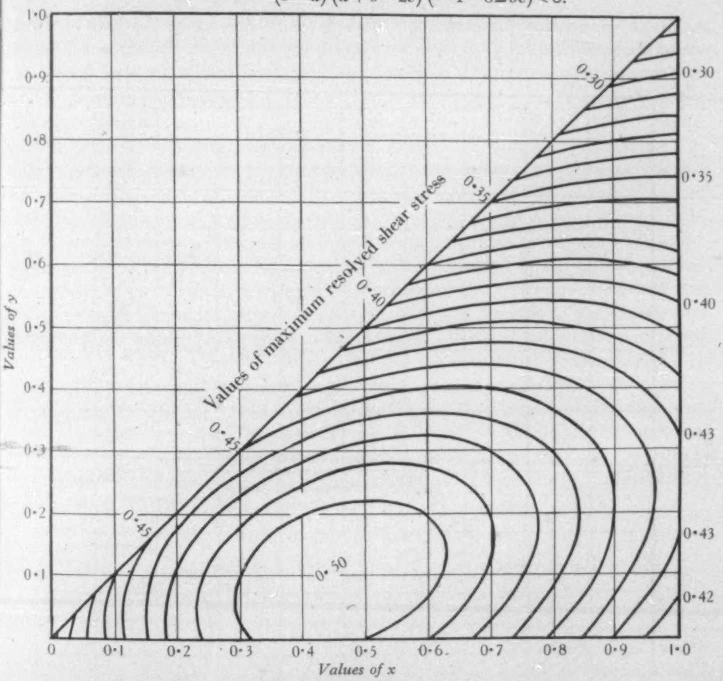


Figure 2

Similarly it may be shown that

$$A^2_{10} < A^2_{03} \qquad A^2_{13} < A^2_{31}$$
$$A^2_{20} < A^2_{02} \text{ or } A^2_{03} \qquad A^2_{21} < A^2_{03} \text{ or } A^2_{12}$$
$$A^2_{30} < A^2_{03} \qquad A^2_{12} < A^2_{03} \text{ or } A^2_{31}$$
$$A^2_{23} < A^2_{32} \qquad A^2_{32} < A^2_{31} \text{ or } A^2_{30}$$

The three coefficients remaining are A_{02}, A_{03} and A_{31}

$$6\,(A^2_{02}-A^2_{03}) = (c-b)\,(b+c-2a)\,(-1-8\Sigma bc) > 0 \text{ if } 2a > b+c, \; < 0 \text{ if } 2a < b+c.$$

$$6\,(A^2_{02}-A^2_{31}) = 4c\,\{4b\,(2a^2+ab+b^2)-a-2b\}.$$

$$6\,(A^2_{03}-A^2_{31}) = 8ac\,(a+b)\,(2b-2c-a)+(b+c)\,(2a-b\mp c)\,(1+8ab).$$

The three equations $A^2_{02} - A^2_{03} = 0$, $A^2_{02} - A^2_{31} = 0$ and $A^2_{03} - A^2_{31} = 0$ define three lines within the elementary triangle that intersect in a common point

$$x\,(=a/c) = \frac{1+\sqrt{51}}{2\sqrt{51}-3}, \quad y\,(=b/c) = \frac{5}{2\sqrt{51}-3},$$

and these lines divide the elementary triangle into three regions, in each of which one of the coefficients A_{02}, A_{03} and A_{31} exceeds the other two. The three regions are shown in figure 3, and in this diagram contours of constant A are also shown.

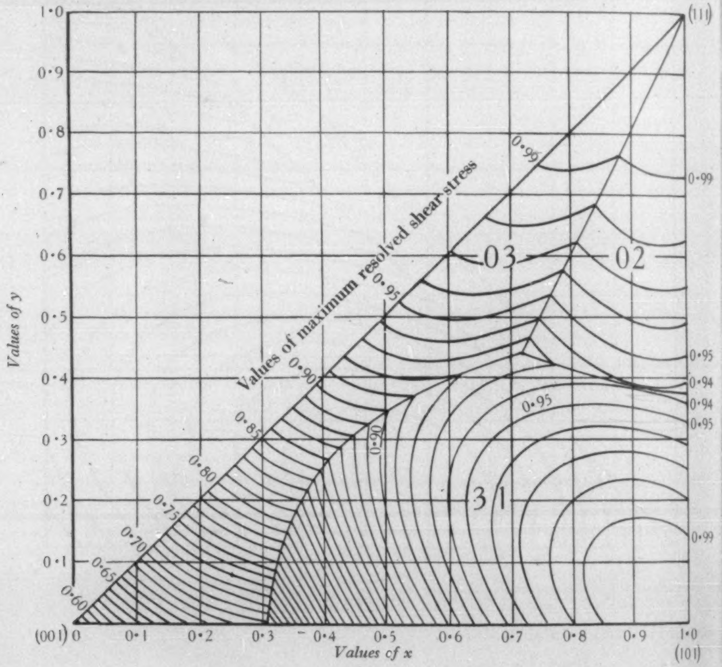


Figure 3

The preparation of a diagram similar to figure 3 for the case of tension is a much simpler matter; for the maximum stress is S_{23} and it is necessary only to calculate contours for this one stress-value. From equation (6) we have

$$S_{23} = T\cos\theta_2\cos\theta_{23} = T\,(a-b+c)\,(b+c)/\sqrt{6}$$

$$= T\,\frac{(1+x-y)\,(1+y)}{\sqrt{6}\,(1+x^2+y^2)} \qquad \ldots\ldots(8),$$

where $x = a/c$, $y = b/c$ as before.

Contours of constant maximum stress are therefore ellipses with parallel axes, the major axis of each ellipse making an angle $\pi/8$ with the line $y=0$. The actual contour map for this case is shown in figure 2.

§ 5. MAXIMUM STRESSES WHEN THE SLIP PLANE MAY BE ANY PLANE THROUGH ANY ONE OF A NUMBER OF CRYSTALLOGRAPHIC DIRECTIONS

The necessary modifications to the analysis given in §§ 3 and 4 to provide for other systems of slip planes and slip directions may be made without difficulty; but in view of the behaviour of single crystals of iron (9, 10, 11), the case in which the slip plane may be any plane through the slip direction is worth special attention. In this case general analysis on the lines of § 3 is scarcely feasible, for the direction cosines of the slip plane are not known. A variable parameter representing rotation of the slip plane about the slip direction must be included, and this parameter will appear in the expression (4), whence its value must be determined to render the value of the whole expression a maximum. The slip plane thus varies not only with λ but also with the relative values of the three stress components Q, S and P, so that expressions similar to (4*a*), (4*b*) and (4*c*) would be of little use. The two more important cases of tension (or bending) and torsion may however be investigated separately.

Again to facilitate the presentation of the analysis, attention will be confined to the case of iron. In this case the slip directions are the normals to the octahedral planes; these directions will be designated 0, 1, 2 and 3 with the same convention as to the position of the specimen-axis as in the case of the face-centred lattice, figure 1. In the case of tension, the maximum resolved shear stress in the direction 0 is given by $S_0 = T\cos\theta_0\cos\theta$ with the condition

$$\sin\theta_0\sin\theta\cos(\psi_0-\psi)+\cos\theta_0\cos\theta=0,$$

where θ, ψ are the co-ordinates of the slip plane. It may easily be shown that

$$S_0=\frac{T\cos\theta_0\sin\theta_0}{\sqrt{\{\cos^2\theta_0\sec^2(\psi_0-\psi)+\sin^2\theta_0\}}} \qquad \ldots\ldots(9).$$

There is no restriction on the value of ψ, so that the maximum value of S_0 is merely $\frac{1}{2}T\sin 2\theta_0$. In this case there is no advantage in expressing the stress coefficients in terms of the co-ordinates (θ, ϕ) of the axis of the specimen (see figure 1); for the lines along which $S_0=S_1$, etc. can be found very easily from the equation

$$\sin 2\theta_0 = \sin 2\theta_1$$

and the side on which $S_0>S_1$ may be determined by inspection. It may easily be shown that the maximum stress is either S_0 or S_2. The line (within the elementary triangle) along which $S_0=S_2$ is given by $2\theta_0=\pi-2\theta_2$, whence $\cos^2\theta_0+\cos^2\theta_2=1$ and, from equation (6), $(1+x+y)^2+(1+x-y)^2=3(1+x^2+y^2)$ or $(x-2)^2+y^2=3$. This is a circle with centre at $x=2$, $y=0$ and radius $\sqrt{3}$. The contours of constant maximum resolved shear stress for the case of tension are shown in figure 4. The contours of constant S_0 are ellipses and hyperbolas and those of constant S_2 are hyperbolas.

In the case of torsion, the stress factors may be expressed in the form

$$S^2_0 = \cos^2\theta_0 \cos^2(\lambda-\psi_0) + \cos^2 2\theta_0 \sin^2(\lambda-\psi_0)^{(11)};$$

but here again there is no advantage in conversion to the co-ordinates (θ, ϕ) of the axis of the specimen. The maximum value of S_0 is either $\cos\theta_0$ or $\cos 2\theta_0$

$$\left[\,|\cos 2\theta_0| > |\cos\theta_0|, \text{ if } \theta_0 > \frac{\pi}{3}\right].^*$$

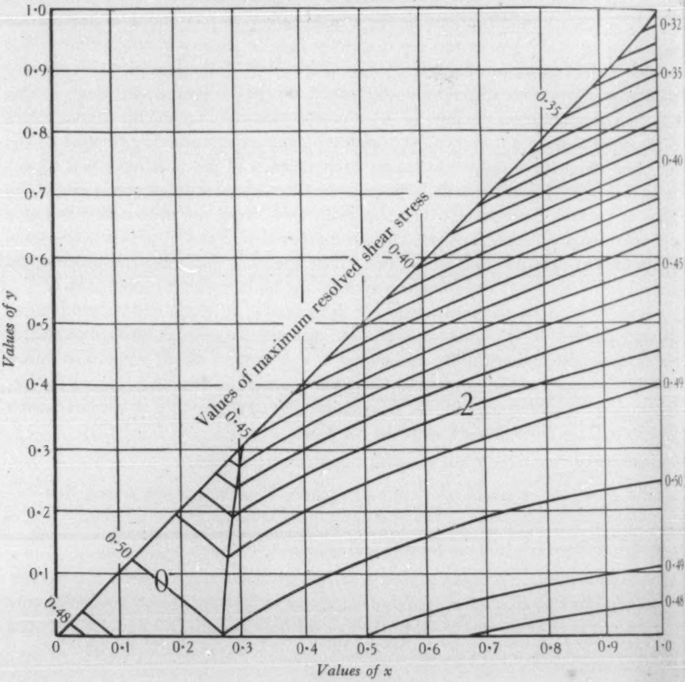


Figure 4

The maximum stress factor for different positions of the axis of the specimen within the elementary triangle, figure 1, may be found by comparison of expressions such as

$$\cos\theta_0 = (a+b+c)/\sqrt{3} \quad\text{and}\quad \cos 2\theta_0 = \tfrac{4}{3}(bc+ca+ab) - \tfrac{1}{3}$$

if we take the greatest numerical value and remember that $c > a > b > 0$, and that

---

* The positive sense of each slip direction is that which makes an acute angle with the positive sense of the axis of the specimen; thus $\theta_0 < \pi/2$.

L

$a^2+b^2+c^2=1$. Thus it may be shown that within the elementary triangle, figure 1, $\cos^2\theta_0$ is greater than $\cos^2\theta_1$, $\cos^2\theta_2$, $\cos^2\theta_3$ and $\cos^2 2\theta_0$, whilst $\cos^2 2\theta_3$ is greater than $\cos^2 2\theta_1$ and $\cos^2 2\theta_2$. The two coefficients to be considered are therefore $\cos^2\theta_0$ and $\cos^2 2\theta_3$. The contours of constant maximum resolved shear stress for the case of torsion are shown in figure 5. It may be noticed that these contours are lines of constant θ_0 or θ_3. All these contours are conics having their centres on the line $x=y$. The contours of constant θ_0 in the upper part of the triangle are ellipses and in the lower part hyperbolas, whilst the contours of constant θ_3 are hyperbolas including the limiting case of two coincident straight lines $(x+y-1)^2=0$. The lines of intersection, where $\cos^2\theta_0=\cos^2 2\theta_3$, are quartic curves; but these also cut the line $x=y$ orthogonally.

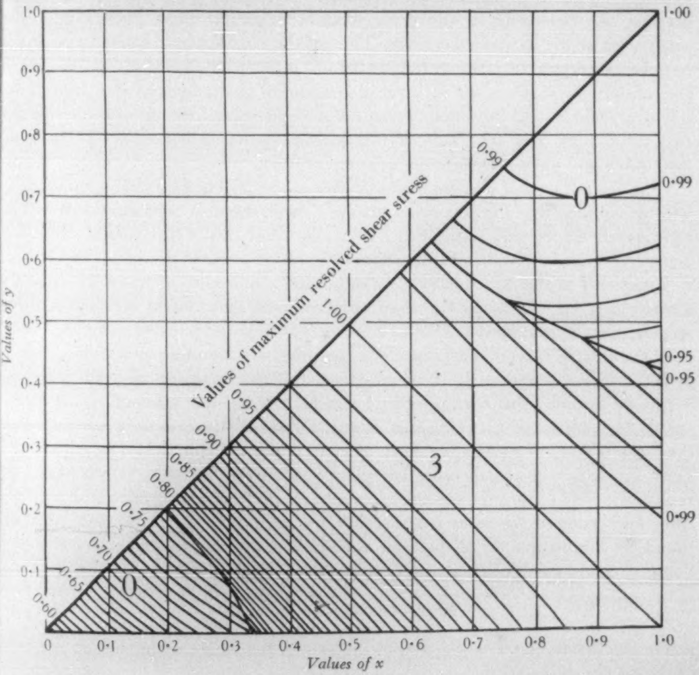


Figure 5

§ 6. ORIENTATION EFFECTS IN POLYCRYSTALLINE SPECIMENS

If a polycrystalline specimen is subjected to any stress system, the behaviour of the specimen as a whole must presumably be determined by the behaviour of the constituent crystals. Apart from any effect due to the intercrystalline boundaries, whether in altering the stress-distribution or in strengthening the individual crystals, the behaviour of the polycrystalline aggregate must depend upon the orientation of the individual crystals. Thus, if the occurrence of some phenomenon in the single crystal (e.g. slip or fatigue failure) is determined entirely by the value of the resolved shear stress, the occurrence of a similar phenomenon in the polycrystalline aggregate should presumably occur at a stress-value intermediate between the two values that give the value of the critical resolved shear stress in the most favourably and most unfavourably oriented crystals.

For the purpose of obtaining an estimate of the effect of varying orientation, it will be assumed that the maximum resolved shear stress in each individual crystal must reach the critical value, but that no increase above this critical value can occur. If fA_m is the value of the maximum resolved shear stress in any crystal due to a tension f in the direction (θ, ϕ), the tension required to give the critical shear stress s is s/A_m, A_m being the maximum resolved shear stress coefficient corresponding to the orientation (θ, ϕ). The total load required to cause complete yield[*] of all the crystals in a cross-section σ of a specimen will therefore be $\Sigma\, s\delta\sigma/A_m$, where $\delta\sigma$ is the area of one crystal. If the orientation of the crystals is perfectly random, it may be assumed that each orientation, and therefore each value of A_m, is represented by a crystal of the same area, so that the mean stress on the cross-section is the mean of s/A_m or s times the mean of $1/A_m$, since s is constant. The ratio of the mean stress f_0 required to cause complete yield to the stress f_m which would be required if yield occurred when the maximum shear stress, irrespective of slip-plane-directions, reached the value s is therefore the mean of $1/A_m$. The mean must be found by summation for all possible orientations within the elementary triangle, figure 1, assuming all orientations to be equally probable. We have therefore

$$f_0/f_m = \text{mean of } 1/A_m = \frac{\iint 1/A_m \sin\theta\, d\theta\, d\phi}{\iint \sin\theta\, d\theta\, d\phi} \qquad \ldots\ldots(10).$$

where the denominator is the area of the spherical triangle within which the axis of the specimen lies.

The value of $1/A_m$ as a function of θ and ϕ for the case of the face-centred lattice can be found from equation (8), but it is more convenient to convert the expression (10) from the co-ordinates (θ, ϕ) to the co-ordinates (x, y). It may thus be shown that

$$f_0/f_m = \frac{6\sqrt{6}}{\pi}\int_0^1 dx \int_0^x \frac{dy}{(1+x-y)(1+y)\sqrt{(1+x^2+y^2)}} \qquad \ldots\ldots(10a).$$

This double integral has been evaluated numerically and gives the result

$$f_0/f_m = 1{\cdot}116 \qquad \ldots\ldots(10b)$$

with a possible error of $\pm 0{\cdot}001$.

---

[*] The sense in which the term "yield" is used will be obvious.

In the case of the body-centred lattice, assuming that slip can occur on any plane through any of the four slip directions, the value of $1/A_m$ cannot conveniently be expressed as a function of θ and ϕ (or x and y), because both the directions 0 and 2 are involved. The mean value of $1/A_m$ over the triangle, figure 4, has therefore been calculated by a process of numerical integration from the plotted contours, the areal distortion of the diagram being of course taken into account. The value thus obtained is

$$f_0/f_m = 1{\cdot}068 \qquad \text{......(11).}$$

with a possible error of $\pm 0{\cdot}002$, where f_0 and f_m have the same meanings as in the previous case.

In the case of torsion, the effect of varying crystal-orientation is rather more difficult to assess. At radius r and angle λ, let the crystal-orientation be (θ, ϕ). Then the value of the maximum stress coefficient A_e depends not only upon θ and ϕ but also upon λ.[*] If T_0 is the torque required to cause complete yield, assuming that the maximum resolved shear stress must everywhere reach the value s, and if T_m is the torque which would be required to cause yield if it were necessary only for the actual maximum shear stress everywhere to reach this same value, the ratio T_0/T_m may be expressed in the form

$$T_0/T_m = \frac{\iiiint s/A_e \sin\theta\, d\theta\, d\phi\,.(r\,d\lambda\,.\,dr)\,.\,r}{\iiiint s \sin\theta\, d\theta\, d\phi\,.(r\,d\lambda\,.\,dr)\,.\,r} \qquad \text{......(12),}$$

or since $1/A_e$ is independent of r

$$T_0/T_m = \frac{\iiint 1/A_e \sin\theta\, d\theta\, d\phi\, d\lambda}{2\pi \iint \sin\theta\, d\theta\, d\phi} \qquad \text{......(12}a\text{).}$$

To evaluate the right-hand side of this equation it is necessary to determine the value of $\frac{1}{2}\pi^{-1}\int 1/A_e\, d\lambda$ for a series of different pairs of values of θ and ϕ, and then to find the average value for all possible orientations.

The value of $\frac{1}{2}\pi^{-1}\int 1/A_e\, d\lambda$ is the mean of the reciprocals of the ordinates of the upper envelope of the usual resolved shear-stress diagram. In the case of the face-centred lattice the evaluation of this mean of the reciprocal of the ordinate involves evaluation of a number of integrals of the form

$$\int_{\beta_1}^{\beta_2} \frac{d\lambda}{A\cos(\lambda-\alpha)},$$

the values of A, α, β_1 and β_2 being first determined. In the case of directed-only slip in the body-centred lattice, the determination of the mean of the reciprocal of the ordinate involves a similar process, but in this case the integrals are elliptic.

The mean value of $1/A_e$ for a number of different orientations has been determined both for the case of the face-centred cubic lattice (aluminium) and for the case of the body-centred lattice (iron, the type of slip plane not being specified). For convenience, the product of the reciprocal of $\frac{1}{2}\pi^{-1}\int 1/A_e\, d\lambda$ and the nominal maximum shear stress has been termed the *harmonic mean stress*. Contours of constant harmonic mean stress for both cases are shown in figures 6 (face-centred lattice) and 7 (body-centred lattice). The value of T_0/T_m has been calculated both for the face-centred and for the body-centred cubic lattices with the following results:

---

[*] A_e is taken as the ratio of the maximum resolved shear stress to the actual maximum shear stress *at the same radius*. Thus A_e is independent of r.

$$\text{for a face-centred cubic lattice, } T_0/T_m = 1{\cdot}281 \pm 0{\cdot}003;$$

$$\text{for a body-centred cubic lattice, } T_0/T_m = 1{\cdot}232 \pm 0{\cdot}003.$$

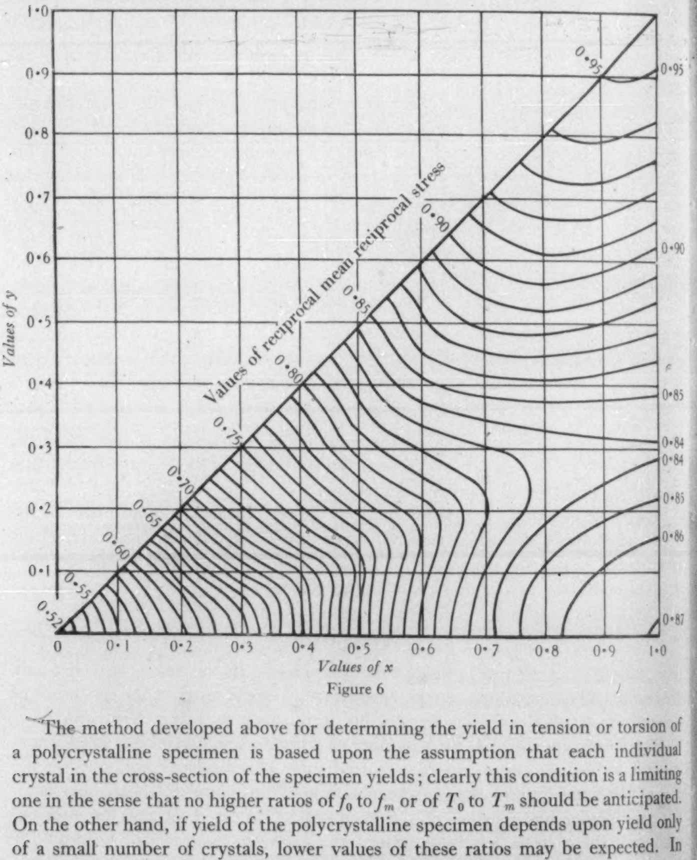


Figure 6

The method developed above for determining the yield in tension or torsion of a polycrystalline specimen is based upon the assumption that each individual crystal in the cross-section of the specimen yields; clearly this condition is a limiting one in the sense that no higher ratios of f_0 to f_m or of T_0 to T_m should be anticipated. On the other hand, if yield of the polycrystalline specimen depends upon yield only of a small number of crystals, lower values of these ratios may be expected. In particular if yield of the specimen is determined by yield of any one individual crystal, the value of both ratios is unity. All possible values of the ratios f_0/f_m and

T_0/T_m lie therefore between unity and 1·116 and 1·281 respectively for the face-centred lattice and between unity and 1·068 and 1·232 respectively for the body-centred lattice.

It is necessary finally to consider whether any intermediate values can be justified by any reasonable assumption that yield of the specimen should depend upon a limited number of crystals intermediate between one and the total number in the cross-section.

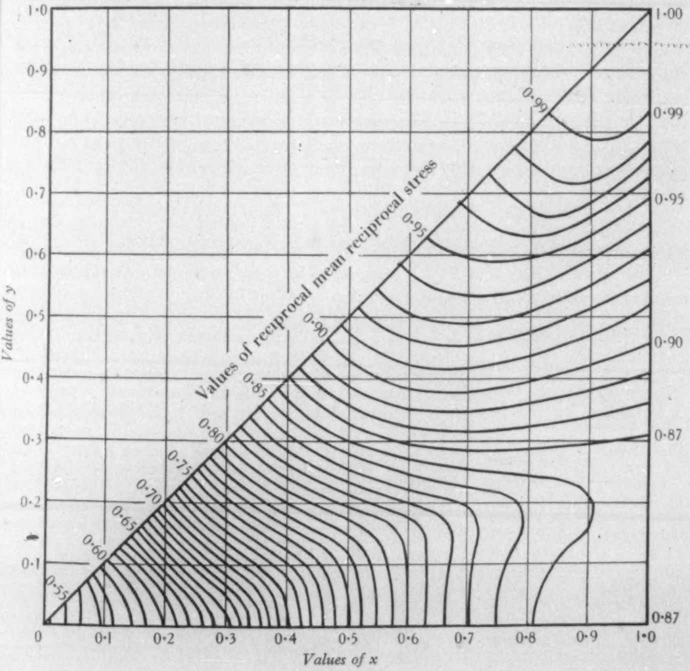


Figure 7

In the case of tension, no other criterion based upon stress conditions alone appears to be justifiable; all the crystals in the cross-section are subjected to the same nominal stress, and there are no logical grounds for dividing the total number into two or more sets. In the case of torsion, on the other hand, the nominal stress (under elastic conditions) is proportional to the radius, so that the stress conditions themselves in some sense differentiate between the surface and the core of the specimen. It is reasonable therefore to consider how the yield of the specimen as a whole would occur if it were determined by yield of all the crystals in the surface at one cross-section. Unfortunately it seems quite impossible to express this criterion in exact terms. Since completely random orientation would give the same (maximum) value of T_0/T_m that was previously obtained, it is clear that the criterion must be based upon some special distribution of orientations of the crystals in the surface; but the sense in which the distribution should be peculiar cannot be defined clearly. There is, however, one possible assumption that may approach reasonably near to the condition required and which has the important advantage that, by its use, the corresponding value of T_0/T_m may be calculated without much difficulty; that is, to assume that, whilst all possible orientations occur at random in the surface, at one cross-section each orientation occurs at the λ-value appropriate to the actual maximum resolved shear stress for that orientation. Under this condition, the corresponding values of T_0/T_m are given merely by the mean reciprocal value of the actual maximum resolved shear stresses, which are shown in figures 2 and 5. These values have been calculated by a process of numerical integration with the following results:

for a face-centred cubic lattice $T_0/T_m = 1{\cdot}133 \pm 0{\cdot}002$;

for a body-centred cubic lattice $T_0/T_m = 1{\cdot}100 \pm 0{\cdot}002$.

The ratios of f_0/f_m and T_0/T_m corresponding to the different criteria are summarized in tables 2 and 3 below.

Table 2. Values of f_0/f_m and T_0/T_m for a face-centred cubic lattice

	Local yield	Surface yield	Complete yield
f_0/f_m	1	—	1·116
T_0/T_m	1	1·133	1·281
Ratio $\dfrac{\text{yield in torsion}}{\text{yield in tension}}$	0·500	0·567 ± 0·001 0·508 ± 0·002	0·574 ± 0·002

Table 3. Values of f_0/f_m and T_0/T_m for a body-centred cubic lattice

	Local yield	Surface yield	Complete yield
f_0/f_m	1	—	1·068
T_0/T_m	1	1·100	1·232
Ratio $\dfrac{\text{yield in torsion}}{\text{yield in tension}}$	0·500	0·550 ± 0·001 0·515 ± 0·002	0·577 ± 0·002

In tables 2 and 3 the values of the ratios of yield in torsion to yield in tension given in the bottom row are obtained merely by dividing T_0/T_m by $2f_0/f_m$. Since the value of f_0/f_m corresponding to the surface-yield value of T_0/T_m cannot be easily decided, in this case the ratio of T_0/T_m to both limiting values of $2f_0/f_m$ has been given.

It is interesting to note that the criterion of local failure gives the ratio 0·500, whereas the criterion of general failure gives the ratio 0·574 in one case and 0·577 in the other. Both these latter values are extremely close to the value $1/\sqrt{3}$ which has been justified mathematically on a different set of assumptions completely independent of the crystalline structure. It is, however, extremely unlikely that the values of these ratios on the assumptions made in this paper should be exactly $1/\sqrt{3}$, although, in view of the approximate nature of the calculations by which the values have been obtained, this possibility cannot be entirely excluded.

If yield or failure of a polycrystalline specimen were determined simply by the maximum stress developed locally in one crystal, the material would certainly be classed as brittle; whereas if yield or failure depended upon yield or failure of each individual crystal, the material would be regarded as ductile. It may therefore be claimed that the effect of variation of orientation of the crystals in a polycrystalline specimen does in itself indicate that the ratio of the yield in torsion to the yield in tension should be 0·500 for brittle materials and about 0·577 for ductile materials.

§ 7. ACKNOWLEDGEMENTS

The work described has been carried out for the Department of Scientific and Industrial Research under the supervision of the Executive Committee of the National Physical Laboratory. The authors wish to express their thanks to Miss W. B. Burns and Mr S. E. Mitchell for considerable assistance in connexion with the calculations, and particularly to Dr H. J. Gough, F.R.S., for the interest he has taken in the work.

REFERENCES

(1) Mark, Polanyi and Schmid. *Z. Phys.* **12**, 58 (1922).
(2) Taylor. Bakerian Lecture, *Proc. roy. Soc.* A, **102**, 643 (1923).
(3) Gough, Hanson and Wright. *Rep. Memor. aero. Res. Comm.*, Lond., No. 995 (1924).
(4) Gough and Cox. *J. Inst. Met.* **45**, No. 1 (1931).
(5) Gough, Wright and Hanson. *J. Inst. Met.* **36**, No. 2 (1926).
(6) Cox and Clenshaw. *Proc. roy. Soc.* A, **149**, 312 (1935).
(7) Gough, Cox and Sopwith. *J. Inst. Met.* **54**, No. 1 (1934).
(8) Wright. *Rep. Memor. aero. Res. Comm.*, Lond., No. 1031 (1926).
(9) Taylor and Elam. *Proc. roy. Soc.* A, **112**, 337 (1926).
(10) Pfeil. *J. Iron Steel Inst.* **15** (1926).
(11) Gough. *Proc. roy. Soc.* A, **118**, 498, 510 (1928).

GLIDE IN METAL SINGLE CRYSTALS

By E. N. da C. ANDRADE, F.R.S.,
Quain Professor of Physics in the University of London
and R. ROSCOE, Ph.D.

Received 7 November 1936. Read 22 January 1937

ABSTRACT. A method is described for producing single crystals of cadmium and of lead which are not affected by annealing, and, in general, show great regularity of mechanical behaviour. A simple recording extensometer also is described. With the cadmium crystals very consistent results have been obtained, and the effects of rate of glide and of impurities on the measured critical shear stress have been investigated. Experiments have been carried out on hardening and recovery of the cadmium crystals, and on the spacing of glide planes in lead crystals, which has been shown to be independent of a range of factors, and so to have a physical significance. It is shown that the results relating to hardening can be explained by a hardening on individual glide lamellae which is proportional to the glide, and it is suggested that the mechanism of the permanent hardening is mainly a rotation of crystallites in the lamellae.

§ 1. INTRODUCTION

The following experiments were carried out in the hope of obtaining information as to the mechanism of the plastic deformation of metal single crystals, and of the significance of the visible slip bands which appear when metal single crystals are deformed. The metals used have been mainly cadmium and lead: cadmium as typical of hexagonal metals, in which, on account of the unique system of glide planes, the phenomena are comparatively simple; and lead on account of the great regularity of behaviour shown by that metal. The work was begun as long ago as 1928 by the senior of us, in conjunction with Dr B. Chalmers, but was held up for some time by the difficulty of obtaining results of sufficient regularity. After the work had been in progress a short time, papers by W. Boas and S. Schmid(1) appeared, which gave some of the results we were in process of obtaining, and so led to a modification of our programme.

We came to the conclusion that much of the irregularity found by previous workers with crystals made under slightly different conditions was due to strain produced during the manufacture of the crystals, and have worked out a new method with the object of producing strain-free single crystals, which, judging from both the low value of the critical shear stress and the consistency of the results, appears to be successful. A simple method of recording photographically the stress-strain relationship is also described. With this we have investigated the question as to whether a finite critical shear stress exists, in particular by varying the initial rate of glide within very wide limits.

The general question of flow, recovery and hardening has been investigated with cadmium, and we have considered, among other things, the effect of small traces of impurity both on the critical shear stress and on the hardening. Our experiments on the spacing of the visible slip bands, in particular with lead, have brought us to consider them as an essential feature of the glide mechanism, and we discuss our results in terms of glide and hardening taking place in laminae separated by distances large compared to their widths.

§ 2. PREPARATION OF THE CRYSTALS

Single crystals of typical metals were first deliberately prepared in 1914(2) by the process of slowly cooling wires which had been heated to the melting point. Since then many other methods have been used, e.g. (*a*) recrystallization of a strained specimen(3), (*b*) crystallization from a point, in tubes or other containing vessels filled with the metal, of which a number of versions have been worked out(4), (*c*) withdrawal from a crucible full of the molten metal of a pointed rod; a crystal forms on the point, which serves as a nucleus on which the single crystal rod forms(5), (*d*) the gradual cooling from one end of a rod actually melted on a flat plate, or in a groove.

In the first three methods it is very difficult not to produce strained crystals, which we particularly wished to avoid. Method (*c*) might at first sight seem to avoid this disadvantage, but actually it does not, judging from the results of Boas and Schmid(1), who found that the behaviour of their crystals depended markedly upon the rate of withdrawal of the rod; the slower the growth the smaller being the critical shear stress. As regards (*d*), in one method, used by Kapitza(6) for bismuth, the rod of metal is placed on a copper plate in which a temperature-gradient is produced by heating one end electrically. The rod is melted, and maintains a cylindrical form due to an oxide coating. Crystallization is then produced by slowly diminishing the heating current. In the method used by Goetz(7) the metal, once more bismuth, is actually melted in a rectangular trough of graphite, arranged with one edge lowermost, so that the rod has an approximately triangular cross-section. A flow of hydrogen is maintained to regulate the thickness of the oxide coat, and the graphite trough, enclosed in an evacuated quartz tube, is slowly drawn through a furnace.

A disadvantage of Kapitza's method is the need of a thick oxide coat, while in Goetz's method the rod produced has an awkward cross-section, especially from the point of view of quantitative measurements of the slip bands.

The method which we have used avoids the thick oxide film, the odd (triangular) cross-section and the possible mechanical disturbances of the crystal due to its motion through the furnace. The loose wire is sealed in a vacuum (about 0·01 mm. of mercury) in a baked out glass capillary tube, which is about twice the diameter of the wire itself. An oxide-free wire so supported can be heated above its melting point and still retain its cylindrical form, except at the ends, where it may run into globules. If melted on a plane surface, it runs entirely into such globules, so that the stability is imparted by the curved solid surface on which it rests.

The crystallization is effected by means of a travelling temperature-gradient. This can be obtained either by using a horizontal furnace with several separate windings (seven, in our case) enclosing the whole length of the tube, or by a very small travelling furnace containing only a short section of the wire at a time. The actual scheme of shunts used with the large furnace is shown in figure 1: by a suitable adjustment of the various resistances it is possible to obtain initially a temperature uniform to within a degree or so all along the furnace. The temperature is measured by means of a thermocouple consisting of two wires, arranged so as to lie in a straight line along the middle of the furnace, as shown in figure 1*a*,

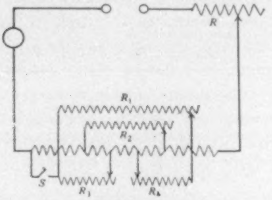


Figure 1.

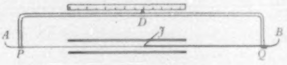


Figure 1*a*.

J being the hot junction. The ends *A* and *B* pass to two cold junctions. The wires are held in a frame *PDQ*, with an indicator at *D*, opposite *J*, by means of which the positions of the hot junction can be read off on a scale. The whole furnace having been adjusted to a temperature of about 15° C. above the melting point, the first section of the winding is short-circuited by means of a key *S*, which produces a steep gradient at the end of the furnace. The resistance R_3 is then gradually diminished by moving the contact of the rheostat at a constant rate, through the agency of a slow-motion geared electric motor. By careful adjustment of the rheostat *R* and of the speed of the motor a constant temperature-gradient and velocity of crystallization can be ensured during the solidification of the whole wire.

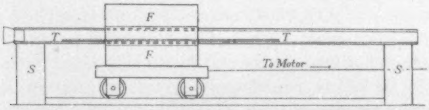


Figure 2.

The method generally adopted, however, is that of the small travelling furnace, which involves the melting of only a short section of the wire at a time (figure 2). Small tubes, each containing a wire, are placed in a hard glass tube of about 1·2 cm. internal diameter which is supported horizontally: the furnace, which fits the tube loosely, is drawn along rails by a slow-motion electric motor.* To ensure steadiness of temperature, the heating current is supplied by accumulators. It is found that, within wide limits, the temperature at a point in the tube depends only on the distance of the furnace from that point, so that a record of the temperature at one point, taken during the whole travel of the furnace, also provides a picture of the distribution of temperature along the tube at any fixed moment. The temperature at one point only was therefore measured with a thermocouple. The temperature-gradient at the point where the wire is melted is controllable; the hotter the furnace the steeper the gradient at the melting point.

---

* This type of instrument, consisting of an ordinary motor provided with heavy reduction gearing, is commercially available, having been designed for slow rotations in shop display.

§ 3. CONDITIONS GOVERNING THE GROWTH OF SINGLE-CRYSTAL WIRES

In order that the molten wire may solidify as a single crystal it is necessary that the crystallization shall take place over a surface which moves along at the same rate as the furnace. Now the temperature-gradient in the wire near to the surface of crystallization is not uniform or of the same form as that in the surrounding glass tube. It suffers a discontinuity at the surface of solidification, its value being lower in the liquid portion than in the surrounding tube and greater in the solid portion, figure 3. This discontinuity is produced by the liberation of the latent heat of fusion of the metal and the difference between the thermal conductivities of the solid and liquid phases. Since the gradient in the liquid can never assume negative values, there is a minimum value for the gradient in the solid at the melting point. If the gradient in the surrounding tube is too low for this to be attained, solidification has to take place over a finite length of the wire which is at the temperature of the melting point, some of the latent heat being carried away by radiation and conduc-

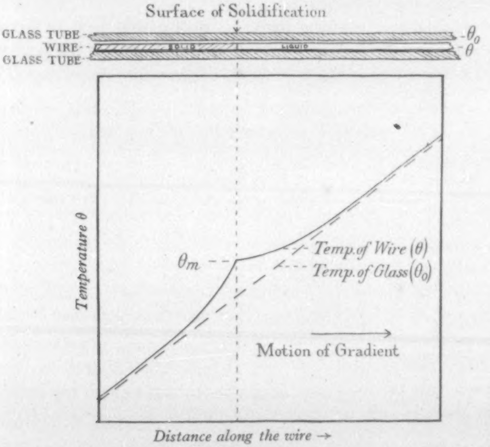


Figure 3.

tion from the surface. In such circumstances crystallization takes place simultaneously at a number of centres, so that growth of single crystals is impossible. By making arbitrary simplifying assumptions, such as uniformity of temperature-gradient in the containing tube, an expression was worked out for the temperature-gradient in the liquid phase. This leads to the condition that, for successful growth,

$$G/V > \frac{1}{1+\sqrt{(K_1/K_2)}} \frac{L}{K_1},$$

where G is the temperature-gradient in the containing tube, V the velocity of travel, K_1, K_2 the thermal conductivities of solid and liquid metal, and L the latent heat per unit volume of the solid. When the numerical values of the constants are inserted we find that G/V must exceed 0·15 deg. hr./cm.^{2} While it has been found experimentally that G/V is the controlling factor for good single-crystal growth, the critical value with our disposition was found experimentally to be about ten or more times as great as that calculated. Since many arbitrary assumptions are made in deriving the formula which, judging from this result, are not justified, the actual deduction is not given here.

The experimental results giving the effect of the ratio G/V on favourable crystal growth are shown in table 1, from which it will be seen that the percentage of good crystals in a batch increases with the ratio G/V, but not with either G or $1/V$, taken separately. In particular, with $G/V = 1·1$ no good crystals were grown. It was also found, however, that the temperature-gradient must not be too large, as is shown by the results given in table 2, where the ratio G/V has a favourable value through-

Table 1. Showing the necessity for a high value for the ratio G/V in growing single crystals

Temperature-gradient G (deg./cm.)	Velocity of growth V (cm./hr.)	Ratio G/V	Number of good crystals in a batch of 20
25	5·0	5·0	18
17	4·3	4·0	20
6	2·3	2·6	14
13	5·0	2·6	12
22	11·0	2·0	10
17	11·0	1·6	10
12	11·0	1·1	0

Table 2. Showing the necessity for a low value of the temperature-gradient G in growing single crystals

Temperature-gradient G (deg./cm.)	Velocity of growth V (cm./hr.)	Ratio G/V	Number of good crystals in a batch of 20
35	11·0	3·2	0
30	11·0	2·7	7
28	10·0	2·8	12
26	3·1	8·4	12
25	5·0	5·0	18
17	4·3	4·0	20

out. These two conditions, $G/V \not< 4{\cdot}0$ deg. hr./cm^2, and $G \not> 20$ deg./cm., were found always to give good results. This means in practice that very low velocities of growth are the most favourable.

With our disposition there was a marked tendency for the growth of cadmium crystals for which the angle χ_0, between hexagonal plane and wire axis, was in the neighbourhood of 15°. This is shown in figure 4, which exhibits the results obtained with 105 crystals, the number having a given value of χ_0 being plotted against χ_0. The reason for this has not been worked out, but it may well lie in the difference of thermal conductivity in, and perpendicular to, the hexagonal plane.

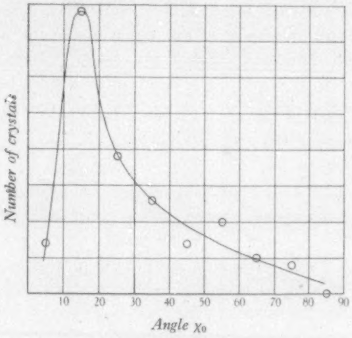


Figure 4.

Very little contamination of the metal occurs in the preparation of crystals by our method. The purest metal used in the course of the work described below was Hilger spectroscopically standardized cadmium, laboratory no. 8961. After a number of rods had been cast and converted into single crystals they were submitted to Messrs Adam Hilger for a second spectroscopic analysis. The estimated amounts of impurity in the original metal and in the finished crystals, as given by the firm, are shown in table 3.

Table 3. Number of parts of impurity present in a million parts of cadmium

Impurity	Hilger cadmium	Crystals
Bismuth	0·25	0·10
Lead	0·15	0·15
Copper	0·50	0·75
Total ...	0·9	1·0

Another quality of cadmium, which is obtainable in the form of wire, was extensively used in this work. On analysis the cadmium was found to contain as much as 0·11 per cent of lead and 0·03 per cent of zinc. This is referred to throughout this paper as the *standard cadmium*, to distinguish it from the very pure *Hilger* *cadmium*. Attempts to produce single crystals having a still higher content of lead (1 per cent) were repeatedly unsuccessful.

§ 4. THE RECORDING EXTENSOMETER

In studying the deformation of single crystals it is desirable to have an instrument which will record the extension and load continuously, the extension ranging up to several centimetres, and the loads being of the order of 100 grams weight. For this purpose the force acting at any moment is measured by the deflection of a metal beam to which the upper end of the wire is attached, a device which makes it unnecessary to measure the actual load applied, which it is often difficult to observe directly. The very small movement of the upper end of the wire makes it easy to control the application of the load. Polanyi and others (8, 9) have already described an extension apparatus which employs the bending of a steel beam to measure the force. Our instrument is simpler in construction, and records automatically.

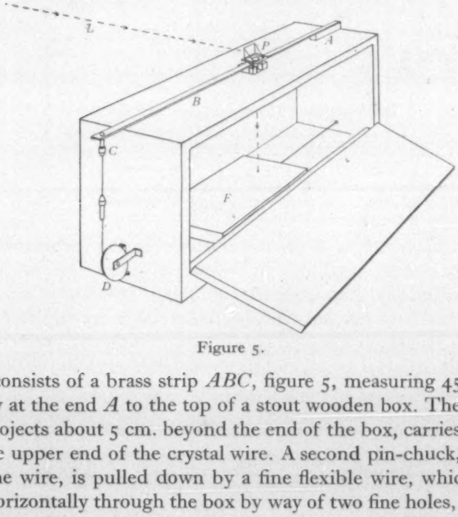


Figure 5.

The beam consists of a brass strip *ABC*, figure 5, measuring 45 × 1·2 × 0·4 cm. and fixed firmly at the end *A* to the top of a stout wooden box. The free end of the beam, which projects about 5 cm. beyond the end of the box, carries a pin-chuck *C*, which holds the upper end of the crystal wire. A second pin-chuck, attached to the lower end of the wire, is pulled down by a fine flexible wire, which passes round the pulley *D*, horizontally through the box by way of two fine holes, and finally over a second pulley, so that tension can be applied by a system of weights. Inside the box the wire is attached to a float *F*, which is carried by a shallow mercury bath. A narrow strip at the bottom retains this mercury, the rest of the opening being closed by a light-tight lid, shown open in figure 5.

The float *F* carries a piece of sensitized paper. On the strip *ABC* at *P* rests one leg of an optical lever, the other two legs of which are rounded at the lower end, one being supported in a groove parallel to the strip, and the other in a triangular depression. Both groove and hole are in brass pieces fixed firmly to the box. The frame of the optical lever carries a right-angle prism. A horizontal beam of light from a pointolite lamp and lens is reflected by this prism through a hole in the top of the box down on to the paper carried by the float, where it is brought to a focus. The angle of the beam of light is always adjusted so that, when the load is increased, for the mean position of the prism the downward beam is truly vertical: this minimizes the deviation from a linear relationship between force and displacement of light-spot. When the end of the wire is loaded, the travel of the float is equal to the extension of the wire, allowance being made for the small elastic yield at the end of the beam, while the displacement of the light-spot measures the applied force. Owing to this yield there is a travel of the float, proportional to the load, even supposing the wire not to stretch at all, and on this account the axes of force and displacement are slightly oblique (see figure 7). For this reason, the single crystal is replaced after the experiment by a practically inextensible polycrystalline wire. A continuously increasing load is then applied, so that the oblique force axis is traced out on the paper. By applying known loads, spots on the paper are then obtained, from which a calibration curve, giving the displacement of the light-spot corresponding to a given load, can be prepared. This turns out to be accurately linear. A displacement of 1 cm. on the paper corresponds to a load of 40 g. wt., and loads up to 300 g. wt. can be used.

The tension is usually applied by a float carried by water in a jar, the level being continuously lowered by a syphon. Although it would be possible, by adjusting the form of the float, to obtain a practically constant rate of extension, this is not worth doing, as it does not ensure that the rate of geometrical glide shall be constant. For more rapid extension it is convenient to let water pass into a light can hanging from the hook: for still more rapid extension, lasting a few seconds, a weight can be applied by withdrawing the hand that holds it. The question of the effect of varying the rate is discussed in § 8, where it is shown that, although gross variations of rate have an important effect, variations of several tens per cent make little difference, so that a precise control is superfluous.

§ 5. DIFFERENT TYPES OF EXTENSION

The general behaviour of hexagonal single crystals under stress has often been described. The glide plane is the hexagonal plane, and the glide direction any one of the digonal axes, the operative one being that lying nearest to the projection of the axis of the wire on the glide plane. Glide takes place when the shear stress per unit area of glide plane, resolved in the direction of glide, attains a critical value. There are, however, various ways in which glide of this kind can manifest itself.

Omitting for the moment the question of double glide and of twinning, we have two main types of extension, due to simple glide. In the one type the wire thins down uniformly along the whole length: in the other type considerable glide takes place locally, over a limited length, before the rest of the wire has extended markedly. In the latter case, as extension proceeds, local thinning, which is accompanied by the usual rotation of the glide planes, ceases, and the adjacent portions of the wire begin to slip. Thus the two discontinuities between the extended and unextended portions of the wire slowly move outwards until they reach the ends, when the wire is uniformly thinned. After this, further extension takes place in a uniform manner. This process has already been described by Boas and Schmid(1). When the glide plane makes a large angle with the direction of pull a still more marked local extension can occur, which consists of a sharply defined glide packet, making a sharp angle with the main wire, like an elbow in a stove pipe, as at (1) in the plate. A picture of this has been given by Boas and Schmid(10), but the phenomenon has not hitherto been systematically discussed.

The cause of the different types of extension lies in the relative importance of the geometrical increase of resolved shear stress and the physical hardening. Owing to the rotation of the planes the resolved shear stress increases during glide. Supposing that the axis of the extended portion coincides with the direction of tension (which, owing to flexual glide, *Biegegleitung*, will be the case when the extended portion is not too short), then

$$d=\frac{l}{l_0}=\frac{\sin\chi_0}{\sin\chi},$$

where l_0 and l are respectively the initial and stretched lengths of the extended portion, to which correspond the angles χ_0 and χ made by the glide planes with the direction of tension. The amount of glide a, defined as the relative displacement of glide planes unit distance apart, is given by

$$a=\frac{1}{\sin\chi_0}\{\sqrt{(d^2-\sin^2\chi_0)}-\cos\chi_0\} \qquad \text{......(1)},$$

if the direction of glide coincides with the projection of the wire axis on the glide plane. (Owing to the three equivalent possible glide directions, this assumption cannot be far from the fact.) Further, with this assumption, the resolved shear stress S is given by

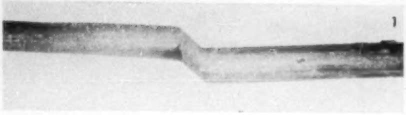
$$S=\frac{W_0\sin\chi_0}{d}\sqrt{(d^2-\sin^2\chi_0)} \qquad \text{......(2)},$$

where W_0 is the load per unit normal cross-section of the unstretched wire. From (1) and (2) a relation between S and a can be worked out for any given value of χ_0. Curves showing this relation for various values of χ_0 are shown in figure 6.

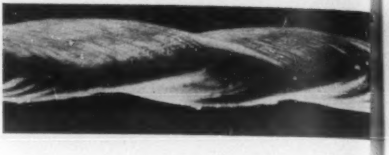
In the same figure is given the experimental hardening curve AE, connecting resolved shear stress with glide. This curve depends somewhat on the rate of extension; see figure 9, from which the curve is taken. Figure 6 corresponds to a rate of increase of shear stress of 0·0204 g. wt. per mm.^{2} per sec.

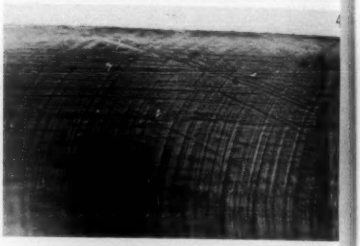
The curves of geometrical increase of stress are given for values of W_0 that correspond to the critical shear stress in each case. It will be seen at once that for small values of χ_0 the slopes of the geometrical hardening curves are less than that of the physical hardening curve, which means that the extension is stable, in the sense that any local slip leads to hardening that will stop the slip until the load is increased. It follows that the wire will extend uniformly. For values of χ_0 greater

PHYSICAL SOCIETY, VOL. 49, PT. 2 (ANDRADE AND ROSCOE)

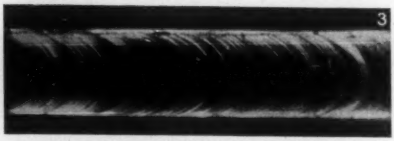

1

× 10


× 25


4

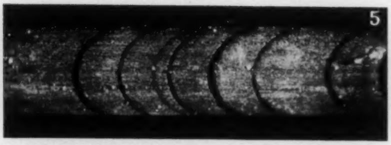
× 200


3

× 25

6

× 800


5

× 25

To face page 161

than about 35°, however, the geometrical increase of critical shear stress exceeds the physical hardening: the process is unstable, and any local glide will continue until a point is reached corresponding to the intersection of the two curves (i.e. at a glide about 2·6 for an initial angle of 60°). Owing to the flexure of the planes at the junction at the slipped and the unslipped portions the effect is rounded off if the angle is not very large (e.g. 45° or 50°). If, however, the angle approaches 90° (say 70° or 80°), the geometrical softening and consequent instability is so great that the stove-pipe effect is produced.

The curves of figure 6 assume that the axis of the wire where glide has taken place lies in the direction of tension. This is, of course, not the case with short glide packets. At first sight the wire to either side of the glide packet at (1) in the plate appears to be straight, and the glide to have taken place without rotation. Actually each portion of the wire has bent, so as to bring the axes of the two end portions of the wire, which are outside the picture, into a straight line, lying along the direction of tension: the bending can be easily verified in the photograph. It is clear that the rotation of the planes in the portion which has experienced glide is far from being sufficient to bring the axis in line with the tension, but there is a small rotation, which is sufficient to produce considerable geometrical softening when, as here, the angle χ_0 is in the neighbourhood of 90°. This is clear from figure 6.

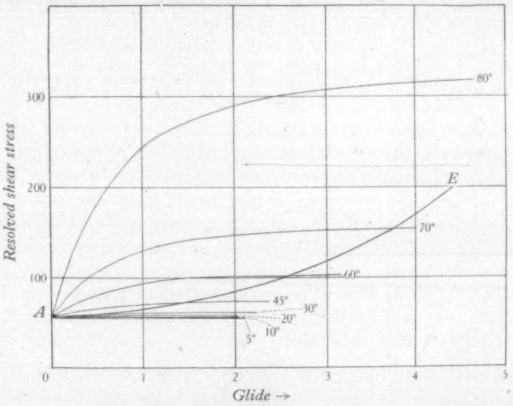


Figure 6.

Curves of the same type have been plotted for a more rapid rate of increase of shear stress, i.e. 8·63 g. wt. per mm^2 per sec., but are not shown here, to save space. They show that the stability is here greater, larger angles being required to produce local glide. This is an observed fact: in particular wires rapidly pulled with the fingers generally give a continuous distribution of glide, even with angles that give the discontinuous type at low rates of stretch.

When local glide takes place it invalidates conclusions drawn from the {shear-stress, glide} relation. Figure 7 shows a trace for a wire which initially showed local glide. The linear drop in the force corresponds to the local thinning, the extension under nearly constant stress to the passage of the discontinuities along the wire, and the final phase of hardening to further extension after the wire has become uniform from end to end.

A particular type of double glide has been observed with cadmium. Although there is only one set of glide planes, there are three equivalent glide directions. In one case two of these were equally favourable, and glide took place in small sections, alternately with one and with the other direction, producing the striking appearance shown at (2) in the plate.

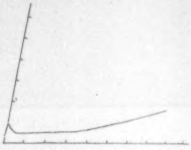


Figure 7.

§6. THE LAW OF RESOLVED SHEAR STRESS

The crystals prepared by the method described above show a very consistent mechanical behaviour, wires with the same orientation of the crystal axes giving the same mechanical strength. In particular, annealing the wires for either a short or a long period produces no change in mechanical behaviour, which points to a very stable condition. This uniform behaviour seems to offer a favourable opportunity of verifying the law of resolved shear stress to a higher degree of accuracy than has hitherto been possible. According to this law, glide begins in a crystal when

$$\frac{F}{A}\sin\chi_0\cos\lambda_0=S_0 \qquad \text{......(3)},$$

F being the load, A the area of normal cross-section, and λ_0 the initial angle between the glide direction and the wire axis, while S_0 is a constant known as the *critical shear stress*.

The values obtained for the load required to produce glide depend somewhat upon the initial rate of glide which results(11). A particular rate (1 per cent per second) was therefore selected, and automatic records of extension against load were made for rates of increase of load which gave an initial rate of glide of this order. Actually, a short period of stretch occurs before the rate of glide becomes uniform, but since this is very small and the corresponding hardening is negligible, it is permissible to take the load corresponding to the point at which the rate first becomes uniform. The orientation of the glide plane and of the glide direction were determined by an optical method described elsewhere(12), and in this way the angle α between the glide direction and the projection of the axis of the wire upon the glide plane was found.

Since $\cos\lambda_0=\cos\chi_0\cos\alpha$, equation (3) may be written

$$\frac{F}{A}\cos\alpha=\frac{2S_0}{\sin 2\chi_0}. \qquad \text{......(4)}.$$

It may be noted that α cannot be greater than 30°, so that $\cos\alpha$ is not far from unity. In figure 8 the experimental points for the standard cadmium, recorded in the following table, are exhibited in a plot of $F\cos\alpha/A$ against χ_0. The curve represents the theoretical equation (4), with the value $S_0 = 55{\cdot}5$ g. wt./mm.² It will be seen that the agreement is remarkably close. The value of the critical shear stress calculated for each crystal is shown in table 4. Taking the average of the two extreme values at each end of the table, a variation of the normal tension $N = (F/A)\sin^2\chi_0$

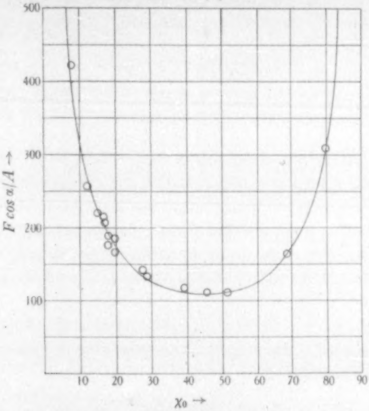


Figure 8.

Table 4. Critical shear stress for standard cadmium (0·11 per cent lead, 0·03 per cent zinc). Rate of glide, 1 per cent per second

Area (mm.²)	Force (g. wt.)	χ_0	α	$\dfrac{F\cos\alpha}{A}$	S_0 (g. wt./mm.²)
0·172	53·2	79·9	6·7	308	52·9
0·170	28·0	68·8	0·0	164	55·8
0·1835	21·8	51·3	20·6	111	54·3
0·1825	22·5	45·7	25·2	112	55·8
0·175	23·8	39·1	30·0	118	57·6
0·181	24·5	28·6	11·0	133	55·9
0·160	24·5	27·5	23·0	141	57·6
0·182	35·9	19·5	19·2	186	58·5
0·185	35·2	19·4	30·0	165	51·7
0·162	30·0	17·8	19·1	175	50·9
0·191	37·2	17·7	15·4	188	54·3
0·171	38·0	16·7	21·4	207	56·8
0·170	39·0	16·4	20·0	215	58·2
0·181	41·0	14·6	14·0	220	56·0
0·179	46·0	11·8	0·0	257	51·5
0·177	75·5	8·2	12·4	422	58·7
				Average value	55·4

from 215 to 10 g. wt./mm.^{2} does not affect the critical shear stress by 2 per cent, which is, of course, well within experimental error. This is probably the most accurate verification of this independence.

It should be mentioned that the critical shear stress for the pure Hilger cadmium is about half this value for the standard cadmium, viz. 26·5 g. wt./mm.^{2} at the same rate of glide.

§7. CRITICAL SHEAR STRESS AND BECKER'S FORMULA

The value for the critical shear stress was obtained by increasing the stress uniformly and noting the value at which plastic yield began, which, as stated in § 6, was found to depend somewhat upon the initial rate of glide. One of us has already published a note on this point(11), and given a curve connecting the critical shear stress with this initial rate: the variation is not great, a tenfold increase of rate changing the critical shear stress by a matter of some 20 per cent only, with the purest cadmium.

The curves suggest some form of logarithmic relation, with log u as a simple function of S_0, the critical shear stress, u being the rate of glide. Such a relation is given by Becker's formula(13, 14),

$$u = Ce^{-V(S_0' - S_0)^2/2GkT} \qquad \text{......(5)},$$

where S_0' is the critical shear stress in the absence of thermal fluctuations, i.e. at absolute zero, S_0 is the actual shear stress, G is the modulus of shear elasticity, k and T have their usual significance, and V is a small volume within which the glide is considered as taking place. Orowan considers that for S_0' and S_0, the macroscopic or experimentally observed values, should be substituted values q times as great, to represent the values in the neighbourhood of the sharp edge of a local crack (*Kerbstelle*), but this clearly contemplates the jump always taking place in a volume V', which is very small compared with the average volume V pertaining to one crack. V is the total volume divided by the number of cracks. To find V, in this latter sense, it seems justifiable to take q equal to 1.

The curve given in Roscoe's paper(11) for Hilger's pure cadmium is well fitted by

$$u \doteq 55e^{-\cdot 00715\,(61\cdot 2 - S_0)^2},$$

u being given as the fractional amount of glide per second; the value 61·2 g./mm.^{2} for S_0' compares well with the 80 g./mm.^{2} obtained by Boas and Schmid(9) at the temperature of liquid hydrogen, considering that we were working with a different specimen of cadmium, and that small impurities have a large effect, as we have established. It may be noted that this formula gives for $S_0 = 17$, 15 and 13 rates of, respectively, 1 per cent glide per 4 minutes, per 13 minutes and per hour. The rate of 1 per cent per hour is a convenient value to adopt as an arbitrary, extremely slow rate, and accordingly we may take $S_0 = 13$ as the critical shear stress.

Since $G = 2 \times 10^{11}$ dynes/cm.^{2} and $kT = 4 \times 10^{-14}$ erg approximately, we have

$$\frac{q^2 V}{2GkT} = 0\cdot 00715 \times 10^{-10} \,(\text{dyne/cm.}^2)^{-2}$$

or

$$q^2 V = 11\cdot 4 \times 10^{-15}$$

which, taking $q = 1$, gives $V = 11\cdot 4 \times 10^{-15}$

or

$$V^{\frac{1}{3}} = 2\cdot 3 \times 10^{-5} \text{ cm.}$$

This is about the value found by Straumanis(15) for the spacing of his layer structure in cadmium crystals, revealed by deposition from the vapour, viz. 8×10^{-5} cm.

§ 8. HARDENING AND RECOVERY

Polanyi and Schmid(16) and Haase and Schmid(17) have carried out experiments on the hardening and recovery of crystals of zinc, tin and bismuth. The former dealt with very small extensions only, of the order of 1 per cent, but the latter extended the considerations to extensions of the order of 200 per cent in the case of zinc. In particular they obtained curves of shear stress against extension at very different rates of extension, the wires being uniformly extended in the Polanyi apparatus. They found that the form of the {shear stress, extension} curve depends upon the rate, the extension corresponding to a given shear stress being greater the slower the rate of extension.

Similar results obtained in the present work on cadmium single crystals are shown in figure 9. The hardening curves I, II and III are the experimental results for the standard cadmium, the curves V and VI for the Hilger cadmium. Each curve represents the mean result for four crystals, and the corresponding rates of application of stress are given beneath the figure, as in these experiments the stress, not the extension, was increased roughly as a linear function of the time. In general the weight was a float sustained in a vessel from which water was withdrawn at a constant rate: strictly speaking this means a uniform increase of load with time, not of resolved shear stress, with the result that the application of stress increases somewhat faster at the end than at the beginning of the extension. As will be seen, however, the curves are not very sensitive to change of rate of application: the differences represented by curves I to III and curves V and VI in figure 9 correspond in each case to changes of rate of the order of a hundred-fold. Hence, changes of rate of increase of stress of tens of per cent produce an altogether negligible effect upon the hardening curves. In the case of the fastest rate the weight had to be applied by hand, with a consequent lack of uniformity, but here again the variations of rate involved are not great enough to affect the curve appreciably.

It has already been remarked that the small amounts of impurities in the standard cadmium produce a marked increase in the critical shear stress necessary to start the deformation. The curves of figure 9 show that this increased shear stress is maintained during the early stages of the deformation, but after a considerable amount of glide (say $a = 4$) has occurred the difference between the curves for the two qualities of cadmium becomes much less pronounced. In order to show this point clearly, curves II*a* and IV have been drawn for the standard cadmium at the same rate of stressing as the curves V and VI for the Hilger cadmium, the ordinates for these rates being calculated by interpolation or extrapolation from the experimental curves. The corresponding curves IV and VI approach one another rapidly as the amount of glide becomes great, while the curves II*a* and V approach one another somewhat less rapidly. It appears, therefore, that when the deformation is very large, impurities (lead and zinc) produce little effect upon the shear stress, particularly when the rate of stressing is high. This would seem to indicate that, when considerable glide has taken place, the resistance to gliding is produced by some factor, such as crystal break-up, which is not important in the early stages of hardening.

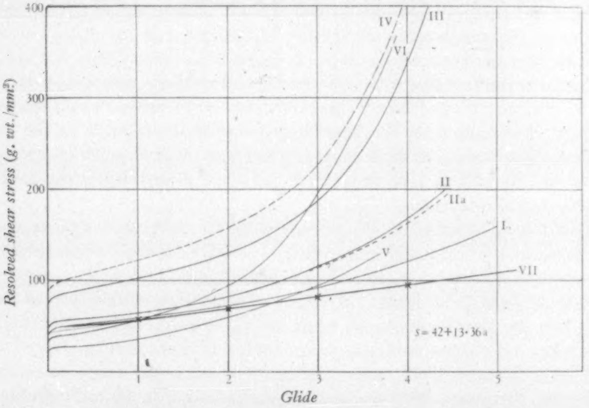


Figure 9. I, II, II*a*, III, IV, our standard cadmium, at rates of increase of shear stress 0·00055, 0·0204, 0·0174, 8·63 and 50·8 g. wt. per mm.² per sec. respectively. II*a* and IV are obtained by interpolation, for comparison with Hilger cadmium. V, VI, Hilger's spectroscopically pure cadmium, at rates 0·0174 and 50·8 respectively. VII, standard cadmium, extrapolated to zero rate.

The dependence of the hardening curves upon the rate of glide or rate of stressing may arise in two different ways which have been carefully distinguished by Orowan$^{(18)}$. Thus Haase and Schmid explain their results by supposing that the low values of the hardening at low rates of stretch are due to a recovery which proceeds simultaneously with the deformation—that is to say they assume that the shear stress is a function of the amount of glide and the time. This is called the "static" conception by Orowan, as opposed to the "dynamic" conception, in which the rate of glide is considered as a function both of the stress and of the amount of glide which has taken place, so that flow under constant stress would be possible even in the absence of recovery.

Orowan has put forward the following modification of Becker's formula for the rate of glide in any part of a crystal which is sufficiently large to be considered as homogeneous for glide purposes:

$$u=\frac{da}{dt}=Ce^{-V(R-qS+ba)^2/2GkT},$$

where u is the rate of glide, a the amount of glide, b and C are constants, R represents the theoretical shear stress in the absence of submicroscopic cracks and temperature fluctuations, S is the externally applied shear stress resolved in the direction of glide, G the shear modulus, k Boltzmann's constant, and V a small volume in which there is a probability that the shear stress may be increased locally to the theoretical value by temperature fluctuations. This is effectively Becker's formula for the critical shear stress, equation (5) in § 7, with the addition of the term ba, which represents a hardening assumed to be proportional to the amount of glide.

At constant temperature this equation gives the hardening S against a as a set of parallel straight lines of slope b/q. When the rate of glide da/dt is maintained constant, dS/dt is also constant. If, therefore, Orowan's formula holds, increasing the resolved shear stress at a fixed rate should produce a constant rate of glide, and the hardening curves obtained by increasing the resolved shear stress at different constant rates should be a set of parallel straight lines. Figure 9 shows that this is not the case at high rates of application of stress and large amounts of glide, although the curves for standard cadmium do form a set of approximately straight lines for amounts of glide up to 200 per cent. It should be remarked in this connexion that Orowan's success has been obtained with extensions of a few per cent only. The mechanism of the great and rapid extension of figure 9 must be essentially different from that contemplated by him, for it may be noted that better agreement cannot be obtained by altering the form of the hardening function ba.

If curves be plotted showing S against log (da/dt) for various values of a, we find that S diminishes as da/dt diminishes, the variation becoming exceedingly small at low values of da/dt. S appears to be tending equally to a constant value for large values of a, although constancy is not actually reached. It is possible, however, to extrapolate so as to get the values of S corresponding to different values of a at a rate of, say, 10^{-5} per cent per sec. Such a limiting $\{S, a\}$ curve is shown as VII in figure 9, the crosses representing the extrapolated values, and it will be seen that it is a straight line. It appears, then, that if various stresses be applied, and if in each case sufficient time be allowed for the length to become constant (for a to reach 4, i.e. an extension of 400 per cent, would require over a year at 10^{-5} per cent per sec.), then there is a straight-line relation between S and a. The variation from the straight line at fast rates is in part due to the crystal structure not having time to take up an equilibrium state. As regards the constancy of a at a given S, which appears to be reached after a long time at these slow rates of increase, it will be observed that the highest value of S contemplated in VII, figure 9 (see § 9) is about 100 g. wt. per mm^2, whereas for the lower curve in figure 11 it is 182 g. wt./mm^2

It is remarkable that while the $(S-a)$ curve approaches a limiting form as the rate becomes very small, there is no sign of a limiting form as the rate becomes very large, but rather, as far as our results go, the effect of rate becomes more and more pronounced as the rates are made larger. If the large values of a which correspond to a given S at very low rates of increase of load were due entirely to a time flow, then we should expect smaller and smaller values of a as we increased the rates, but when the rate of load became rapid we should expect to find a limiting value of a corresponding to no flow. It appears, however, that even at high rates an increase of rate still leads to a more rapid rise of the $(S-a)$ curve. This is inexplicable on the static theory.

The reason why, as shown, the Becker-Orowan theory does not apply to these experiments is no doubt to be sought in the rapid rates. The underlying basis of the theory is that within the metal there exists cracks or dislocations in the neighbourhood of which the local stress, produced by the macroscopic load, has nearly the theoretical value. As a result of thermal agitation, the local stress attains from time to time the theoretical value somewhere in the volume V, and glide takes place. The mechanism by which this occurs may be that, put forward by Dehlinger(19), of the propagation of the dislocation. The energy required to raise the stress from S to R is $(R-qS)^2/2G$, if the substance is supposed to be behaving elastically in the neighbourhood of the local irregularity. If, further, each elementary glide yields the same contribution to the macroscopic glide, then there follows at once the formula

$$\log\frac{da}{dt}=\log C-\frac{(R-qS)^2}{2GkT}V \qquad \ldots\ldots(6)$$

$$=\log C-M\,(R/q-S)^2 \text{ if } T \text{ is constant.}$$

The formula appears to hold for certain cases of slow glide, but it clearly cannot hold in extreme cases. If $S=R/q$, for instance, a state which is easily attained, the rate has a maximum value. This is quite contrary to observation and is obviously impossible, so that S must not be taken too large. Another way of reaching the same conclusion is to notice that the theory contemplates static conditions in the neighbourhood of the dislocation, so that elastic strain can be assumed. It is clear that with very rapid glide there is no approach to equilibrium conditions, and the expression for the energy taken in equation (6) is not justified.

To obtain favourable conditions for real recovery we extended a crystal of Hilger cadmium by a very large amount very quickly, the glide being over 300 per cent in 3 sec. When the extension is carried out as rapidly as this, the form of the {stress, glide} curve is insensitive to the exact rate of increase of load, as has already been indicated. After a shear of 334 g. wt./mm.^{2} had been reached the load was taken off for half an hour and the wire was then reloaded to 334 g. wt./mm.^{2} in 3 sec., after which a half-hour period of recovery was again given. This process was repeated, the loading always being carried out in 3 sec. and half an hour allowed for recovery, until the crystal twinned and broke. Figure 10 represents the result. It will be seen that the parts EF, HK, MN, PQ and RS of the {stress, glide} curves are repetitions of the portion BC of the original curve, with the same slope, displaced to the right, so as to correspond to a larger glide in each case. This part, then, represents real recovery, the wire after a period of rest behaving exactly as it did at a lesser glide before rest. The portions *DE*, *GH*, *LM*, *OP*, which represent a spurious recovery due to constant-stress flow in the original curve, become smaller for each successive loading, so that the apparent recovery *QR* tends to be entirely real recovery.

The method of very rapid loading appears to be suitable for the elucidation of the law of hardening and recovery. These experiments serve to establish, then, that while the form of the $(S-a)$ curve is largely influenced by the fact that under the stresses used there is a flow with time, so that the curve depends on the rate, yet there is a large real recovery after even large extension. The separation of the two effects has been clearly effected.

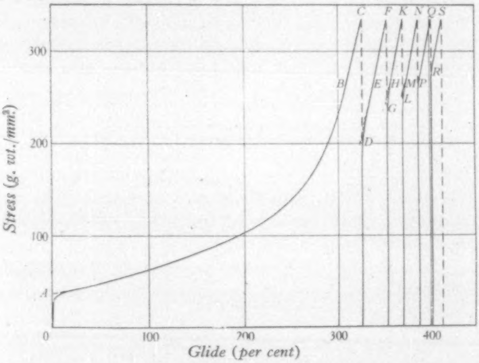


Figure 10. Extension of cadmium single crystal loaded to a stress of 334 g. wt./mm² in 3 sec. Half-hour recovery intervals.

The position may be summed up as follows. When the rate of glide is very slow, or when the deformation is not very large, the hardening obeys a linear law, and Orowan's theory, considered in conjunction with the idea of recovery, gives an adequate explanation of the dependence of hardening upon rate. With high rates and large extensions another type of hardening appears which is permanent, i.e. has no recovery, although it depends on the rate to a marked extent. This latter type of hardening depends much less upon the presence of small amounts of impurities than does the former type.

§ 9. FLOW UNDER CONSTANT SHEAR STRESS

When a large load is applied to a single-crystal wire of cadmium, an immediate extension takes place which is followed by a creep continuously decreasing in rapidity, until at last the crystal twins and fractures. With a large load the extension may increase measurably for several weeks, without any sign that a final length is being asymptotically approached.

With a sufficiently large constant load, the glide planes are almost immediately brought so close to the axis of the wire that the resolved shear stress can be treated as constant throughout the flow. The form of the {extension, time} curve is then determined by the rate of flow alone, which is itself a function of the constant stress and the extension. In these experiments a simple method of optical projection, giving a magnification of about 5 times, was used: greater magnification led to no advantage, on account of the jerky nature of the stretch which was then revealed.

The flow curves, figure 11, obtained in this way show a very rapid decrease in the rate of flow u, which, however, exhibits no tendency to become vanishingly small. In fact, when log u is plotted against $1/a$ a straight line results, so that

$$\frac{da}{dt} = u = A \,.\, e^{c/a} \qquad \ldots\ldots(7).$$

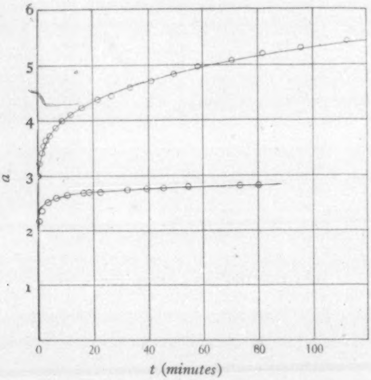


Figure 11.

The points on the curves of figure 11 were obtained with the standard cadmium; Hilger cadmium crystals give the same type of curve. The upper curve corresponds to a shear stress of 282 g. wt./mm^2 and the lower to 182 g. wt./mm^2. The constants in the above formula come out to be, for the upper curve $A = 3{\cdot}27 \times 10^{-5}$ per min., $c = 29{\cdot}0$; and, for the lower curve, $A = 4{\cdot}79 \times 10^{-12}$ per min., $c = 64{\cdot}5$.

The following relation is obtained from equation (7):

$$t = \frac{1}{A}\int_0^a e^{-c/a}\, da \qquad \ldots\ldots(8).$$

By using the above constants and calculating the value of this integral for various values of the upper limit, the continuous curves of figure 11 were obtained. The circles, representing the observed values, lie very close to the curves. Other formulae with two constants which were tried gave a very much worse representation.

Equation (7) gives a very different {flow, extension} curve to the Becker-Orowan formula. In particular, when $a=\infty$, u has a finite value, while it should become zero according to the Becker-Orowan formula. It has been shown in § 8 that the latter formula is valid under constant rate of stressing when the rate of stressing is low and the extension small. For large rate and extension, however, there is an abnormal hardening which produces a *lower* value of u than that given by the Becker-Orowan formula. The finite value of u at $a=\infty$ given by equation (7) must therefore be due to a real recovery persisting even at very great extensions. Thus

$$u=\frac{da}{dt}=-\left(\frac{da}{dS}\right)_t\left(\frac{dS}{dt}\right)_a,$$

where $(dS/da)_t$ represents the coefficient of hardening with increasing glide, extrapolated to zero time—corresponding to the b of Orowan's formula, which has been shown in § 8 to be constant—and $-(dS/dt)_a$ represents the true recovery.

§ 10. THE SPACING OF THE SLIP BANDS

The slip bands show irregularity of spacing which has hitherto discouraged measurement(20). With cadmium, slip bands easily visible to the naked eye are seen as soon as a small percentage extension is produced, and become more prominent as extension proceeds. A wire extended by several per cent is shown at (3) on the plate. With a high-power microscope, giving a magnification of 1000, these coarse slip bands resolve themselves into exceedingly close lines, or finer slip bands. This is best seen with the ring type of vertical illumination. With the same methods of observation very close lines have been detected all along the wire, as shown at (4) on the plate. Although a certain regularity in the spacing of the lines is observable at a magnification of 1000, alteration of the illumination brings up a series of still fainter lines between the regularly spaced set. The separation of the planes shown at (4) is about $1{\cdot}5\times10^{-4}$ cm. The separation of the faintest bands detected varies from about 10 to 5×10^{-5} cm., but this limit may be imposed by the resolving-power of the microscope. With great extension the distinction between the different classes of bands becomes more marked: a cadmium wire extended by 400 per cent, with a few very marked bands, is shown at (5). The rest of the wire is covered with fine bands not distinguishable in the photograph.

The most regular results were obtained with lead, where the distinction between prominent and minor bands is very clear. Bands in lead, extended by about 30 per cent are shown at (6); between these are some very much fainter bands not distinguishable in the photograph. This clearly marked nature of the prominent bands encouraged measurement of the separation. Figure 12 shows the distribution of about 120 successive intervals. It will be seen that the form resembles the standard-error curve, with a most probable separation of 17 arbitrary units, which corresponds to a separation (measured normal to the planes) of $4{\cdot}2\mu$.

To see if this spacing had any physical meaning it was measured under a variety of conditions, the diameter of the wire, the temperature at which the extension took place, the rate of extension and the amount of extension being varied. The results are shown in table 5, from which it appears that, within the limits tested, none of the factors has any effect on the spacing. It seems, then, that the spacing is a real property of the metal or, at any rate, of a particular specimen of metal. The effect of impurities in the metal has not been investigated.

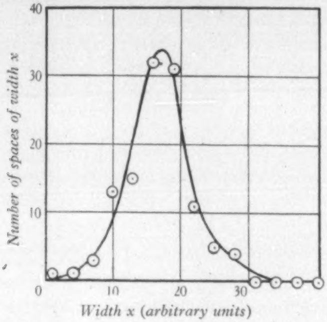


Figure 12.

Table 5. Spacing of glide planes of single crystals of lead

Factor	Magnitude	Mean spacing of glide planes (μ.)
Extension	15 per cent	3·85
	24 ,,	4·27
Rate of deformation	0·001 sec.$^{-1}$	4·31
	0·003 ,,	4·01
	3·0 ,,	4·13
Diameter of wire	0·091 cm.	4·29
	0·045 ,,	4·18
Temperature	15 °C.	4·20
	100 ,,	4·27
	Mean	4·17

In the case of lead an interesting phenomenon was observed which is shown at (6) on the plate. Some of the slip bands did not run completely round the wire, but stopped short opposite one another. This is strongly reminiscent of G. I. Taylor's theory of plastic slip, where he shows that two dislocations of opposite sign will, under stress, move in opposite directions until they come abreast. To make them pass one another the stress has to be increased, and reaches a maximum when the distance by which they overlap equals the separation, measured normally to the path of dislocation. Further increase of stress produces instability, and the dislocations run on indefinitely in opposite directions. Now it is to be observed that in no case were bands overlapping by more than the distance of separation observed, and that, while the ends of the bands are not sufficiently definite for exact measurement, yet the overlap of more widely separated bands is rather more than that of less widely separated bands; see in particular the third, fourth, fifth, sixth and seventh bands from the left in photograph 6 on the plate. This tends to show that the bands are really the paths of dislocations propagated somewhat in the Taylor manner.

§ 11. DISCUSSION OF RESULTS

The results here described do something, it is hoped, to clear up certain obscurities regarding the flow and hardening of single crystals. The preliminary work on the production of strain-free crystals was necessary before a sufficient uniformity of behaviour could be obtained, and the confirmation of the law of resolved shear stress, which is more precise than any previously recorded, seems to show that the crystals were, macroscopically speaking, uniform in structure. It is to be observed that annealing of these crystals, whether for a long or short time, had no effect on their mechanical properties.

As regards the critical shear stress, it has been shown to be dependent on two factors, the initial rate of flow, and the purity of the material. The initial rate can be connected with the critical shear stress by an exponential formula of the Becker type, which gives a finite rate for any shear stress, however small, but nevertheless the form of function is such that the critical shear stress varies extremely slowly with the rate, and an arbitrary small rate of 1 per cent per hour has been selected, which gives $S_0 = 13$ g. wt./mm.^{2} for spectroscopically pure cadmium, which is about one quarter the value given by Schmid$^{(21)}$ (58 g. wt./cm.^{2}).

In our discussion of hardening we have distinguished the following types: (1) a hardening proportional, with cadmium, to the amount of glide, when the rate of loading tends to zero, although the extension may be considerable. It also holds with high rate, if the extension is very small. This is non-recoverable, and corresponds to a simple case discussed by Orowan$^{(22)}$. (2) With large extensions and high rates of loading we have (*a*) a hardening which increases rapidly with increasing glide and is not subject to recovery, and (*b*) a hardening subject to recovery. The amount of this latter hardening decreases with increasing extension.

We believe that any attempted explanation must take into account the fact that the glide takes place, not uniformly on all equivalent crystallographic planes, but in the limited regions known as glide planes. It has been shown here that in the case of lead, in which definite measurements can be made, the separation of these planes is independent of a wide variation of various physical factors, which points to their having a real significance. The general question is complicated by the different behaviour shown by different metals as regards the appearance of the glide planes. With cadmium, speaking generally, all the planes appear early, and increasing extension takes place by further glide on the planes already developed. With this metal the hardening is, in the simple limiting case where the rate is not involved, proportional to the glide. With aluminium, on the other hand, Yamaguchi$^{(23)}$ has found that new glide planes continually appear between the old as the stretching force is increased, the average distance between the planes being approximately inversely as the force. With this metal a parabolic law of hardening holds, to which G. I. Taylor has directed special attention(24) as a case of particular interest. If, as is here suggested, there is a linear hardening with glide on a single glide plane (which could perhaps be better described as a single glide lamella) in the case where a time flow is not involved, then this law can be readily explained, supposing that the glide planes extend right across the material. For if N is the number of developed glide planes per unit distance, measured normal to the planes, then, according to Yamaguchi

$$N = C(S - S_0),$$

while on our assumption

$$S - S_0 = Bx,$$

where x is the glide in a lamella, B and C being constants.

$$N\,dS = B\Sigma\,dx = B\,da,$$

where a is the total glide, so that

$$C(S - S_0)\,dS = B\,da$$

or

$$S - S_0 = A\sqrt{a},$$

which is Taylor's formula. Whether the different behaviour as regards the appearance of glide planes is governed by the temperature, referred to the melting point, is a question which we hope to investigate later.

As regards the development of glide planes, it seems likely that slip takes place by the propagation of a dislocation from a surface flaw, as illustrated by Orowan(25). That the surface plays a prominent part is illustrated by Roscoe's observations on the large increase of critical shear stress caused by an extremely thin surface film of oxide(11). The propagation of a dislocation, considered as a region of self-strain, has been fully discussed by Taylor, who considers that the travel is stopped by a second series of flaws(24), lying at right angles to the direction of propagation. The exact mechanism of the stoppage is not here considered. It is here assumed, however, that the metal contains, as well as the surface cracks, internal flaws of various degrees of gravity—let us say, for instance, cracks, parallel to the glide planes, of elliptical cross-section and of various radii of greatest curvature—from which dislocations can be propagated. To explain hardening it must be supposed that, with a given force, glide takes place from flaws, which are associated with an enhanced local stress. The regions of high stress are disturbed by the passage of the original dislocation, and, when these are exhausted, flaws where the local stress is not so high can be brought into operation by increasing the force.

This supposes that when a region of dislocation or self-strain approaches flaws of different degrees of severity or local curvatures, the yield tends to take place in the direction of a line joining the region of self-strain to the region of greatest shear stress, that is, towards the most severe flaw, distances being equal. The distribution of stress in such cases has not been worked out, but it may be possible to investigate it by the method of photoelasticity.

Plastic flow can, then, be tentatively explained by considering the visible glide planes as localities of intensive glide, proceeding in the first instance from surface flaws, of the Griffiths type, and involving later less grave local internal flaws. The very marked non-recoverable hardening which takes place under rapid flow may be attributed to the locking action of small fragments of crystal that have rotated. Such localized "crystal break-up", as Gough calls it, has been experimentally proved with aluminium single crystals and with mild steel(26, 27). It has been objected to this view that hard fragments cannot lock soft material, but this objection is based on a misunderstanding. The suggestion is that the general unflawed lattice is strong, but contains flaws which are a source of weakness, from which dislocations are propagated. If the soft regions, which lead to visible glide lamellae, are made hard, the whole crystal becomes hard. When the flow is very slow, so that equilibrium is slowly attained, the exhaustion of first the weak flaws, and then the stronger ones, leads to progressive hardening on the lamellae, but more rapid stressing can lead to actual rotation of fragments, or, in Burgers' terminology(28), local curvatures of glide lamellae. Such rotated crystallites are of unflawed material, and so offer a very effective barrier to the propagation of dislocation. G. I. Taylor(29) has pointed out that, in the case of a face-centred cubic crystal, for instance, however two neighbouring fragments are oriented, the ratio of the values of the components of the maximum resolved shear stress of each crystallite cannot exceed 2 to 1, from which he concludes that rotation cannot lead to much hardening. If, however, the function of the rotated crystallites is to hinder glide proceeding from flaws, then the case considered by Taylor does not apply.

When two of Taylor's dislocations, a positive and a negative, travelling in opposite directions, come opposite to one another there is a turning moment on the substance between them which rises to a maximum: when the normal distance between the paths is very small this moment rises to a high value, and may, it is suggested, lead to the rotation of the crystal fragment.

The recoverable part of the hardening may possibly be connected with the other type of crystal distortion, the existence of which has been proved by Gough and Wood, namely dislocated grains, in which there are changes of direction up to 2°.

The effect of impurity is of some interest. It has been shown that traces of foreign metal increase the critical shear stress considerably, which may be attributed to the impurities segregating to the flaws, and making their edges less sharp. When considerable flow has taken place, however, the behaviour of the contaminated tends to be the same as that of the uncontaminated metal, and this is easily understood if the mechanism of hardening at this stage is mainly that of crystallite rotation.

All these suggestions, some of which have something in common with a theory sketched by Burgers(30), are recognised as being of a tentative nature: further development is postponed in the hope of obtaining more definite experimental evidence.

REFERENCES

(1) Boas, W. and Schmid, E. *Z. Phys.* **54**, 16 (1929); **61**, 767 (1930).
(2) Andrade, E. N. da C. *Phil. Mag.* **27**, 869 (1914).
(3) Carpenter, H. C. H. and Elam, C. F. *Proc. roy. Soc.* A, **100**, 329 (1921).
(4) Tammann, G. *Lehrbuch der Metallographie* (Leipzig, 1923).
Bridgman, P. W. *Proc. Amer. Acad.* **60**, 305 (1925).
Hausser, K. W. and Scholz, P. *Wiss. Veröff. Siemens-Konz.* **5**, 144 (1927).
(5) Czochralski, J. *Z. phys. Chem.* **92**, 219 (1917).
(6) Kapitza. *Proc. roy. Soc.* A, **119**, 358 (1928).
(7) Goetz. *Phys. Rev.* **35**, 193 (1930).
(8) Polanyi. *Z. tech. Phys.* **6**, 121 (1925).
(9) Boas and Schmid. *Z. Phys.* **61**, 767 (1930).
(10) Boas and Schmid. *Z. Phys.* **57**, 575 (1929).
(11) Roscoe, R. *Phil. Mag.* **21**, 399 (1936).
(12) Roscoe, R. and Hutchings, P. J. *Phil. Mag.* **16**, 703 (1933).
(13) Becker, R. *Phys. Z.* **26**, 919 (1925).
(14) Orowan, E. *Z. Phys.* **89**, 605 (1934).
(15) Straumanis, M. *Z. phys. Chem.* B, **13**, 316 (1931).
(16) Polanyi and Schmid. *Z. Phys.* **32**, 684 (1925).
(17) Haase and Schmid. *Z. Phys.* **33**, 413 (1925).
(18) Orowan. *Z. Phys.* **89**, 605, 614 and 634 (1935).
(19) Dehlinger, U. *Ann. Phys., Lpz.*, **2**, 749 (1929).
(20) Schmid and Boas. *Kristallplastizität*, p. 157 (Berlin, 1935).
(21) Schmid. *International Conference on Physics*, **2**, 165 (1934).
(22) Orowan. *Z. Phys.* **89**, 614 (1934).
(23) Yamaguchi. *Inst. Phys. Chem. Res.* Tokio, No. 151, 289 (1928).
(24) Taylor, G. I. *Proc. roy. Soc.* A, **145**, 362 and 388 (1934).
(25) Orowan. *Schweizer Arch.* Heft 7, **5**, figure 7 (1935).
(26) Taylor, G. I. and Gough, H. J. quoted in *Marburg Lecture*, p. 51 (1933).
(27) Gough, H. J. and Wood, W. A. *Proc. roy. Soc.* A, **154**, 510 (1936).
(28) Burgers, W. G. *International Conference on Physics*, **2**, 139 (1934).
(29) Taylor, G. I. *Trans. Faraday Soc.* **24**, 121 (1928).
(30) Burgers, W. G. *Report on Viscosity and Plasticity* (Academy of Sciences, Amsterdam, 1935).

DISCUSSION

Prof. W. Wilson asked whether there is an elastic region within which small stresses fail to produce permanent deformation or glide, and what factors determine the thickness of the glide bands.

Prof. A. F. C. Pollard asked whether ring illumination, as distinct from critical illumination, had been used for illustration no. 6 in the plate. A photomicrograph with critical illumination might yield an even more beautiful result. Presumably the lines in the illustration were the ill-defined images of the exposed and narrow portions of the slip planes. The two boundaries of such a portion might be resolvable in some places on the crystal surface. The appearance of the two lines in the bottom left-hand corner of the illustration seemed to indicate this.

Dr B. Chalmers. It would be interesting to know whether the authors attach any importance to the bend that occurs in the {stress, glide} curves of figures 9 and 10 in the region corresponding to the early stages of glide. It appears that this gradual commencement of glide may be connected with the apparent variation of critical shear stress with rate of application of the stress. The critical shear stress, if defined as the smallest stress that causes glide, cannot depend on the rate of stressing, although its measurement may depend on the sensitivity of the extensometer; if, however, it is defined as the stress that will produce glide at some arbitrary speed, then it will of course depend on this speed.

I should like to know whether the true critical shear stress, as calculated for slow rates, bears any relation to the stress at the first observable point of departure of the {stress, glide} line from the stress axis.

I should like to mention in connexion with the statement on p. 170 of the paper about the jerky nature of the stretch that in experiments on single crystals of tin, in which much higher magnification but slower rates were used, I have detected no jerkiness or irregularity at all.

AUTHORS' reply. In reply to Prof. W. Wilson: There may be an elastic region within which small stresses fail to produce permanent deformation, but our methods were not designed to deal with stresses and strains as minute as those here involved. There is no satisfactory explanation of the observed separation of the regions of intensive glide.

In reply to Prof. A. F. Pollard: The method of critical illumination has been applied to the observation of slip bands, but, in our hands, did not appear to give results sensibly better than those obtained with the ring illumination. In view of some work on slip bands in solid mercury carried out by Dr K. M. Greenland (shortly to be published) in the laboratory at University College it would appear possible that the surface exhibits continuous curvature across the boundary, and that the double line represents reflections from the two curved edges of the band.

In reply to Dr B. Chalmers: Dr Chalmers remarks on the dependence of critical shear stress on rate of glide represent the point of view that we have endeavoured to put forward in the paper. The critical shear stress is defined not as the smallest stress which causes glide, since this is indefinite if our views are correct, but as the stress which causes glide to set in at a given rate. The rate chosen as characteristic is that corresponding to the approximately straight portion of the {stress, glide} curve, which sets in almost immediately after loading (see Figures 9 and 10), and so corresponds to the extrapolated point *A* in Figure 10. As we have pointed out, the stress is not sensitive to small changes of rate, so that the latter need not be accurately determined. The jerky nature of the deformation which occurred when the crystals were stretched under constant stress may possibly be due to slight vibration, due to chance causes, of the support to which the upper end of the crystals were affixed, which would produce fluctuation in the stress acting upon the crystals.

X-RAY INVESTIGATION OF PURE IRON-NICKEL ALLOYS. PART 2: THERMAL EXPANSION OF SOME FURTHER ALLOYS

By Professor E. A. OWEN, M.A., D.Sc., F.Inst.P.

AND

E. L. YATES, M.Sc., Ph.D., A.Inst.P.

University College of North Wales, Bangor

ABSTRACT. The lattice parameters of three iron-nickel alloys containing respectively 41·1, 32·2 and 24·2 per cent of nickel by weight have been measured at different temperatures. The alloys had to be carefully heat-treated before any spectra could be obtained with them. From the relations between parameter-value and temperature, curves were obtained showing how the coefficient of thermal expansion varied with temperature. The two alloys containing 41·1 and 32·2 per cent of nickel yielded curves which showed that the coefficient of thermal expansion had a somewhat low value at low temperatures. At a particular temperature the value increased suddenly to another value, which on further rise of temperature remained almost constant. These results are in agreement with the results obtained by other observers by other methods of measurement.

The expansion curve of an alloy containing 24·2 per cent of nickel showed the ordinary thermal-hysteresis effect found with the irreversible alloys. The thermal expansion of the components α and γ were measured. The mean atomic volume in the α phase is greater than the mean atomic volume in the γ phase at the same temperature. By the aid of these measurements it was possible to explain the form of the expansion curve obtained with a rod of the material, which shows a contraction when the α phase transforms to the γ phase.

Values are given for the lattice parameters of the pure α and γ phases at different temperatures. The coefficient of thermal expansion of the body-centred α phase is approximately $8{\cdot}2 \times 10^{-6}$ at 0° C. and $14{\cdot}0 \times 10^{-6}$ at 200° C., and that of the face-centred γ and γ' phases is $17{\cdot}5 \times 10^{-6}$ between 0° C. and 630° C.

§ 1. INTRODUCTORY

In a previous paper(1) iron-nickel alloys in the γ' phase containing high percentages of nickel were examined. Two of the alloys dealt with in this paper are also in the γ' phase but contain more iron than those previously considered, being not far removed from the iron-rich boundary of the γ' region. They are of interest because they have compositions on either side of that of invar, which contains about 36 per cent of nickel. The third alloy is in the region which is usually regarded as a mixed phase of α and γ'. It was of particular interest to study the thermal expansion of this alloy by the X-ray method, in view of the somewhat anomalous behaviour of alloys in this region of composition, as observed by Chevenard and others in measurements of their thermal expansions by the ordinary methods.

The alloys were marked 43, 32 and 24 and their compositions are given in table 1.

Table 1. Composition of alloys

Alloy	Nickel content	
	Weight (per cent)	Atoms (per cent)
43	42·7	41·7
32	32·2	31·1
24	24·2	23·3

§ 2. HEAT TREATMENT OF THE ALLOYS

The general experimental procedure was similar to that adopted with the alloys considered in the previous paper, but individual attention had to be given to the heat treatment of the alloys. It was necessary, as with the other alloys, to cool them very slowly after they had been annealed either in lump or in powder form. The heat treatments to which the alloys were submitted are summarized in table 2.

Table 2. Heat treatment of alloys

Alloy no.	Specimen no.	Lump annealing			Powder annealing		
		Temperature (° C.)	Time (days)	Period of cooling (days)	Temperature (° C.)	Time (hours)	Period of cooling (hours)
43	1	800	10	4½	600	14	48
	2	800	10	4½	600	12	129
32	1	800	12	4½	600	14	48
	2	800	12	4½	600	12	129
	3	800	12	4½	600	15	225
24	1	—	—	—	500	1	—
	2	—	—	—	300	24	7
	3	—	—	—	900	1	48
	4	—	—	—	600	15	Quenched
	5	800	12	4½	600	14	48
	6	800	21½	6	600	17	384

§ 3. RESULTS

Alloy 43. Little difficulty was experienced with this alloy in obtaining good reflections. Powder cooled from 600° C. in two days gave lines as well defined as those obtained with a specimen of the alloy cooled from the same temperature in five days. Cobalt* radiation was found suitable for use with this alloy as the lines were registered on the film in positions corresponding to the biggest glancing angle possible with the camera and therefore were capable of high accuracy of measurement. This radiation also yielded photographs with clear background, thus greatly facilitating the measurement of the lines. Exposures at temperatures above 500° C.

---

* The wave-lengths taken were Cobalt $K\alpha_1$ (1·78529) and Cobalt $K\alpha_2$ (1·78919).

gave rather weak lines which would have been difficult to measure with high accuracy were it not that the background was comparatively free from fogging.

The alloy possessed a face-centred cubic structure and the lines measured in the photographs were doublets reflected from the (400) planes. The results of the measurements are summarized in table 3; the letters (*c*) and (*h*) which appear in this table have the same significance as they had in our previous paper$^{(1)}$.

Table 3. Lattice parameters of alloy 43 at different temperatures

Film no.	Temperature (° C.)	Lattice parameter (A.)	Film no.	Temperature (° C.)	Lattice parameter (A.)
E 398	10	$3\cdot5876_5$	E 404	401	$3\cdot5958_2$
E 419 & E 423	12	$3\cdot5878_6$	E 415	422	$3\cdot5967_1$
E 411 & E 416	13	$3\cdot5879_4$	E 421	446 (*h*)	$3\cdot5975_6$
E 410, E 429, E 434	14	$3\cdot5878_4$	E 435	457	$3\cdot5987_0$
E 405 & E 418	15	$3\cdot5879_2$	E 417	478	$3\cdot5996_4$
E 402 & E 426	16	$3\cdot5878_3$	E 425	481 (*h*)	$3\cdot5995_9$
E 413	17	$3\cdot5878_4$	E 430	489	$3\cdot6004_3$
E 400	102 (*c*)	$3\cdot5893_8$	E 437	495 (*h*)	$3\cdot6005_2$
E 401	200 (*c*)	$3\cdot5912_3$	E 440	501	$3\cdot6011_0$
E 399	303	$3\cdot5928_2$	E 431	511	$3\cdot6012_6$
E 414	328	$3\cdot5933_2$	E 424	524	$3\cdot6019_1$
E 420	346 (*c*)	$3\cdot5936_0$	E 432	537 (*c*)	$3\cdot6030_0$
E 409	360	$3\cdot5943_3$	E 438	572	$3\cdot6043_4$
E 412	383	$3\cdot5948_9$	E 406	604	$3\cdot6069_5$

The parameter increases linearly with temperature up to about 300° C. The lattice then expands rapidly until at about 425° C. the parameter again increases linearly with temperature. The variation of the lattice parameter with temperature is shown in figure 1, from which the values of the coefficients of expansion given in table 4 were obtained.

Table 4. Coefficients of thermal expansion of alloy 43

Temperature (° C.)	Lattice parameter (A.)	True coefficient of expansion ($\times 10^6$)
0	$3\cdot5876_5$	4·7
100	$3\cdot5893_3$	4·7
200	$3\cdot5910_0$	4·7
300	$3\cdot5927_2$	5·4
350	$3\cdot5939_0$	7·4
400	$3\cdot5956_0$	12·1
500	$3\cdot6009_2$	15·0
600	$3\cdot6067_3$	15·1

Alloy 32. This alloy is just inside the pure γ' phase and the structure should therefore be face-centred cubic. Spectrum photographs of the alloy taken with copper radiation indicated this structure.

Difficulty was experienced in obtaining well-defined spectral lines with this alloy. A powder (specimen 1, table 2) which had been annealed for 14 hours at

600° C., on being cooled from this temperature to room-temperature in two days, gave a broad diffuse band instead of a doublet. The same heat treatment followed by a period of cooling extending over 129 hours yielded a resolved doublet, but the individual lines of the doublet were still too broad. The background was also rather dark and this made measurement of the lines difficult. To reduce the intensity of the background, vanadium radiation was substituted for the copper radiation hitherto used with this alloy. A third specimen was given the same preliminary heat treatment in lump and in powder form as the other two specimens, but was allowed 225 hours to cool from 600° C. to room-temperature. Well defined lines were now obtained in photographs taken with the specimen maintained at temperatures above 200° C., but below this temperature the definition of the lines was still not good. The lines obtained at the lower temperatures were, however, resolved and were situated in positions on the film where they could be fairly accurately measured. It would appear from the broadening of the spectral lines below about 200° C. that the lattice breaks up or becomes distorted when the specimen is cooled to lower temperatures at the rate which was adopted. This matter needs further investigation.

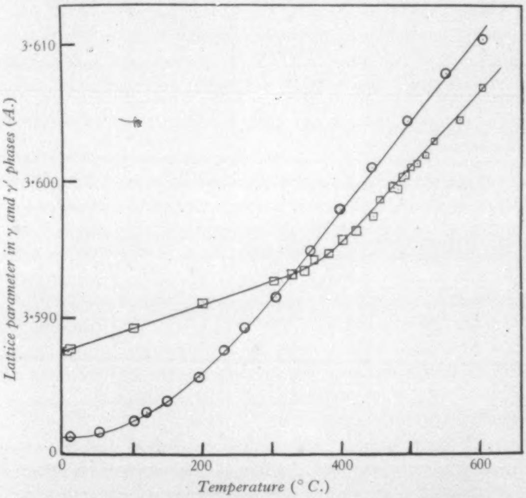


Figure 1. ⊙ 32·2 per cent of nickel; ⊡ 42·7 per cent of nickel.

The lattice parameters recorded in table 5 were measured over the full range of temperatures at which the camera could be used. The powder was changed frequently when the photographs were taken in order to avoid errors due to contamination in the enclosure.

Table 5. Lattice parameters of alloy 32 at different temperatures

Film No.	Temperature (° C.)	Lattice parameter (A.)	Film no.	Temperature (° C.)	Lattice parameter (A.)
E 453 & E 467	12	$3\cdot5814_1$	E 464	305	$3\cdot5915_3$
E 448 & E 451	15	$3\cdot5813_3$	E 469	328	$3\cdot5933_5$
E 454	54	$3\cdot5816_6$	E 452	357	$3\cdot5949_8$
E 455	103 (*c*)	$3\cdot5826_4$	E 461	400 (*c*)	$3\cdot5981_2$
E 456	120 (*c*)	$3\cdot5831_8$	E 465	445 (*h*)	$3\cdot6011_5$
E 450	150 (*c*)	$3\cdot5840_1$	E 462	497 (*c*)	$3\cdot6044_9$
E 457	195 (*h*)	$3\cdot5858_0$	E 468	555 (*h*)	$3\cdot6080_4$
E 466	230 (*c*)	$3\cdot5878_0$	E 460	606 (*h*)	$3\cdot6104_6$
E 449	261 (*h*)	$3\cdot5893_8$			

These results are included in figure 1, from which by drawing tangents to the curve at different temperatures the true coefficients of expansion collected in table 6 were calculated.

Table 6. Coefficients of thermal expansion of alloy 32

Temperature (° C.)	Lattice parameter (A.)	True coefficient of thermal expansion × 10^6
0	$3\cdot5811_0$	2·2
100	$3\cdot5825_7$	5·7
200	$3\cdot5860_7$	14·3
300	$3\cdot5916_8$	16·6
400	$3\cdot5979_0$	17·3
500	$3\cdot6042_0$	17·6
600	$3\cdot6105_8$	17·8

The curves in figure 2 show how the true coefficient of linear expansion varies with the temperature in the alloys examined. The dotted curve was obtained by Chevenard(2) from measurements made on a rod of an alloy containing 35·4 per cent of nickel. This curve closely resembles those now obtained from measurements of the crystal lattices of the alloys. Initially the coefficient of thermal expansion increases very slowly with temperature, but at a certain temperature which increases as the

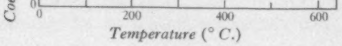


Figure 2.

nickel content of the alloy increases, a sudden rise in the value of the coefficient of expansion takes place, the actual increase in the value being the greater, the lower the nickel content of the alloy. After the sudden rise which is confined to a comparatively narrow range of temperature, the coefficient of expansion on further rise of temperature shows little increase.

The mean temperatures of the ranges in which the coefficient of expansion changes rapidly with temperature for alloys 43 and 32 are respectively about 375° and 150° C., and therefore agree approximately with the temperatures of magnetic transformation, namely 360° and 150° C., for these two alloys. Hence the coefficient of thermal expansion of these two alloys changes from a low to a high value when the temperature passes through that corresponding to the magnetic transformation. This result differs from that obtained with the alloys rich in nickel reported in our previous paper, for which it was found that the coefficient of thermal expansion gradually increased as the temperature was raised. It reached a maximum value at the transformation temperature and suddenly fell to a lower value on passing through this temperature.

The phenomenon now observed with alloys 43 and 32 is similar to that which occurs with β brass when the temperature of the alloy passes through the β-transformation temperature, the thermal coefficient above the β-transformation temperature being greater than that below it. It is to be noted that the coefficient of thermal expansion for both alloys 43 and 32 is abnormally small at 0° C., whereas at temperatures above 400° C. the coefficient is large and greater than that of either nickel or iron.

Alloy 24. Considerable difficulty was experienced in obtaining X-ray reflections which could be measured with this alloy. Six specimens were examined, the heat treatment in each case being different; see table 2.

The annealing given to specimen 1 was considered to be insufficient to remove the distortion produced in the lattice by the cold work done on the material. It was desirable to examine both the phases that were supposed to be present at temperatures not exceeding about 345° C. in this region of composition. The second specimen was consequently annealed at 300° C. for 24 hours and cooled slowly over 7 hours. Very broad faint lines were obtained. A measurement of these lines showed that they were due to the body-centred α phase. No evidence of the face-centred γ phase was obtained with the sample in this condition. The α-phase lines were better defined in a photograph taken with the sample maintained at 306° C., but when the temperature was raised to 632° C. a pair of lines at a smaller arc were recorded on the film. These were due to reflections from the (400) planes of the face-centred γ-phase structure. A photograph taken after the sample had been allowed to cool to room-temperature showed neither α- nor γ-phase lines. When the specimen was heated to 598° C. the γ-phase lines reappeared but they were rather faint.

According to some equilibrium diagrams of this alloy system, the γ phase transforms to $(\alpha+\gamma)$ at 345° C. Three successive exposures were made with specimen 2 at 400° C., the first after 10 min. heating, the second after $1\frac{1}{4}$ hours' heating and the third after heating for a further hour at 400° C. Exposures of 30 min. were given to each photograph so that the powder had received nearly 3½ hours' annealing at 400° C. when the final exposure was made. This annealing was not sufficient to transform the α phase, for the very poor lines in the first photograph remained as a faint band in the last photograph.

On taking a fresh sample of powder and exposing at 603° C. the γ-phase lines again appeared. This same specimen was then heated at 405° C. for 10 hours before being exposed at that temperature and again the γ-phase lines appeared. When it was cooled, however, to room-temperature the α phase did not return.

In order to improve the definition of the lines, some filings were annealed at 900° C. and slowly cooled over 48 hours (specimen 3). An exposure at room-temperature yielded a very faint trace of lines, the arcs corresponding to which were somewhat smaller than those obtained with specimens 1 and 2. These lines gave a lattice parameter of 2·860 A.

As slow cooling appeared to have an adverse effect on the intensity of the lines, quenching from 600° C. was tried (specimen 4) but no lines resulted.

The best results were obtained with specimen 2, which had been annealed at 300° C. for 24 hours and cooled from this temperature to room-temperature in 7 hours. This heat treatment was adopted before the exposures at different temperatures were made. Cobalt radiation was used. Although 26 exposures were made, only 10 of them could be measured with sufficient accuracy to yield reliable results. These are collected in table 7; they are not as accurate as those obtained with the other alloys because the lines were not so well defined. The variation of the lattice parameter of the body-centred α phase with temperature is shown in the top curve of figure 3.

Table 7. Lattice parameters of α phase at different temperatures

Film no.	Temperature (° C.)	Lattice parameter (A.)	Atomic volume (A.^{3})
E 274	15	2·8627	11·730
E 249	16	2·8634	11·739
E 272	98	2·8657	11·767
E 273	198	2·8685	11·801
E 271	293	2·8714	11·837
E 250	306	2·8724	11·850
E 260	340	2·8738	11·867
E 266	405	3·5999	11·663
E 254	598	3·6118	11·779
E 251	632	3·6141	11·802

About 18 months after these preliminary experiments had been made with alloy 24, powder was prepared from an ingot which had been lump-annealed at 800° C. for 21½ days and allowed to cool for 6 days to room-temperature. The powder was annealed at 600° C. for 17 hours and cooled to room-temperature over a period of 384 hours (specimen 6). Photographs taken with specimens of this powder at different temperatures exhibited the lines of the face-centred γ phase. Reflections were obtained from the (310) planes when manganese radiation was used. The lines at room-temperature were broad, and only an approximate value of the lattice parameter could be calculated from them. When heated to 600° C., however, the alloy yielded a pair of resolved lines which could be accurately measured. It was noticed that the lattice parameter of the γ phase at these high temperatures was in close agreement with the values previously obtained for the γ phase when cobalt radiation was employed. There is no structural change in crossing the boundary between the γ' and γ phases, and the lattice expands uniformly from the one phase into the other in alloys in this region of composition.

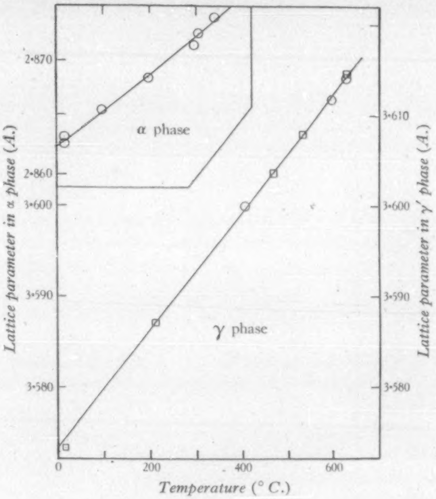


Figure 3.

The results obtained with manganese radiation are recorded in table 8 and are shown in the lower curve of figure 3. In this figure, the parameters of the body-centred α phase and the face-centred γ phase have been plotted on different scales

Table 8. Lattice parameters of γ phase at different temperatures

Film no.	Temperature (° C.)	Lattice parameter (A.)	Atomic volume (A^3)
E 649 & E 651	15	3·5757	11·426
E 650	210	3·5874	11·542
E 646	404	3·6000	11·664
E 649	466	3·6036	11·699
E 645	532	3·6079	11·741
E 647	633	3·6146	11·807

so as to show the relation between the atomic volumes in the two structures. The coefficient of expansion of the body-centred α phase is approximately $10\cdot1 \times 10^{-6}$ at 0° C., $10\cdot3 \times 10^{-6}$ at 100° C., $11\cdot1_5 \times 10^{-6}$ at 200° C. and $13\cdot2 \times 10^{-6}$ at 300° C., while that of the face-centred γ (or γ') phase has a constant value of $18\cdot2 \times 10^{-6}$ between 0° C. and 630° C.

Without a knowledge of the exact proportions of the α and γ phases in the alloy it is impossible to calculate what its mean expansion should be. Hanson and Hanson(3) give the boundaries of the $(\alpha + \gamma')$ region at 9 and 32 per cent of nickel. Assuming that alloy 24, which contains 24·2 per cent of nickel by weight, is made up of 33 per cent of α and 66 per cent of γ, we have calculated with the figures given in tables 7 and 8 how the mean atomic volume should vary with temperature. The mean atomic volume on this basis is 11·52 A^3 at room-temperature, 11·70 A^3 at 340° C., 11·66 A^3 at 400° C., and 11·78 A^3 at 600° C. Thus on the disappearance of the α phase the mean atomic volume of the alloy decreases, and hence a rod of the material would show a contraction at or near this temperature. The results based on the above assumption as to the relative amounts of the α and γ phases in alloy 24 are shown graphically in figure 4, and inset is a curve obtained by Chevenard with an iron-nickel alloy containing 25·9 per cent nickel by weight. The latter shows a contraction between 425° C. and 475° C. and is a curve similar to that now obtained from parameter measurements of the constituent phases. On being cooled to room-temperature the material used by Chevenard contracted uniformly until its dimensions were about 7 per cent smaller than they had originally been at this temperature. This behaviour is characteristic of the irreversible iron-nickel alloys. A bar of an irreversible alloy on being heated, expands up to a certain temperature and then contracts over a range of temperature before expanding again on being further

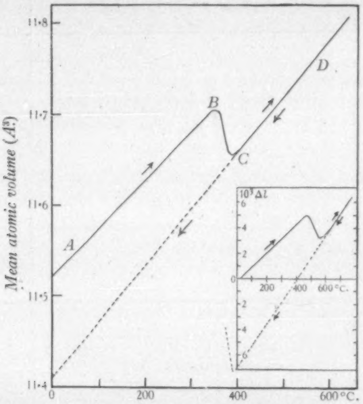


Figure 4. Curve of mean atomic volume for an alloy containing 24·2 per cent nickel by weight, computed on the basis of one-third α phase and two-thirds γ phase.

heated. When cooled from this temperature it contracts uniformly through the temperature at which anomalous contraction took place on heating.

The ordinary methods of measuring thermal expansions do not allow of the same analysis as is possible with the X-ray method of measurement. In the latter it is possible, in the manner described in this paper, to follow the changes in the lattices of the two constituents of the alloy with change in temperature. It is clear from this investigation that the anomalous contraction which occurs at certain temperatures when the irreversible iron-nickel alloys are heated is due to the disappearance of the body-centred or α constituent, the mean atomic volume in which is greater than that in the γ phase at the same temperature. When the alloy is cooled the α phase does not make its appearance at the temperature at which it disappeared on heating, with the result that the alloy now contracts along the pure γ-phase curve to a lower temperature before the α phase appears, thus showing a thermal-hysteresis effect. The temperature at which the α phase appears when the alloy is cooled depends upon the rate of cooling of the alloy.

The difference in the slopes of the portions *AB* and *CD* of the curve shown in figure 4 is due to the fact that the coefficient of thermal expansion of the α phase is smaller than that of the γ phase.

§ 4. CONCLUSIONS

The curves obtained by the X-ray method of measurement described in this paper are similar in form to those obtained with these alloys by previous workers who used different methods of measurement; in fact there is very close agreement between the results obtained by the different methods employed.

It is found that alloys in the region of the γ' phase behave differently according to whether they are in the iron or in the nickel end of the region. In the one case the coefficient of thermal expansion increases suddenly from a low, approximately constant, value to a higher value which shows slight increase on further rise of temperature; in the other case it gradually increases as the temperature is raised, and after reaching a maximum value it suddenly drops on further rise of temperature to a value which remains approximately constant at higher temperatures. The mean temperatures of the sudden rise or the sudden fall in the value of the coefficient correspond to the temperatures of magnetic transformation of the alloys.

The crystal structure on both sides of the magnetic transformation remains the same, but the difference in the behaviour of the alloys as they pass through the temperature of magnetic transformation would seem to point to a radical difference between them.

The boundary of magnetic transformation, that is the boundary of the γ' phase, is usually shown in equilibrium diagrams of this alloy series. Since the crystal structure of the alloys is the same on both sides of the boundary it might be argued that there is no need to include the boundary in the diagram, but the changes that take place in the values of the coefficient of thermal expansion when the temperature passes through the transformation temperature seem to afford sufficient ground for including the boundary in the diagram were there no other reasons for keeping it. The structure of β brass remains the same when the alloy passes through the transformation temperature, but its coefficient of thermal expansion changes in the same manner as do those of alloys 41 and 32 when they pass through the temperature of magnetic transformation. The alloy in the copper-zinc diagram is shown as passing from the β' to the β phase. Similar instances could be cited, all of which indicate that the boundary of magnetic transformation should be shown in the equilibrium diagram.

The results described in this paper point rather to the need for differentiation between the two limbs of the boundary, since it is found that the alloys in passing through the transformation temperature behave differently according as they are of composition near the nickel or near the iron end of the γ' phase.

Alloy 24 is interesting because, unlike the other alloys so far investigated, it is in the mixed $(\alpha+\gamma')$ region at room-temperature. The results described in the paper provide an explanation for the shape of the expansion curves obtained with alloys in this region of composition. They are the irreversible alloys which show thermal hysteresis. This phenomenon is observed because the alloy, which is usually in rod form when its expansion is measured, is not allowed sufficient time to reach the true equilibrium state corresponding to the temperatures at which the observations are made. When an irreversible alloy is heated, the α phase disappears at a comparatively low temperature and the γ phase exists alone at temperatures above this. On cooling the γ phase persists alone at temperatures much below that at which the α phase disappeared on heating. Owing to these facts, a composite bar of the material behaves in an abnormal manner. It shows a contraction in the neighbourhood of the temperature at which the α phase disappears.

By the X-ray method the various phases present can be independently examined, and the results throw light on the shape of the curves obtained from measurements on a composite bar of the alloy in a manner which the ordinary methods of measurement would not render possible.

§ 5. ACKNOWLEDGEMENT

The authors wish to record their indebtedness to Dr L. B. Pfeil of the Mond Nickel Company for his advice and for kindly preparing the alloys and supplying their compositions.

REFERENCES

(1) Owen and Yates. *Proc. phys. Soc.* **49**, 17 (1937).
(2) Chevenard. *Comptes Rendus*, **159**, 177 (1914).
(3) Hanson and Hanson. *J. Iron and Steel Inst.* **102**, 39 (1920).

A PHOTOELECTRIC SPECTROPHOTOMETER OF HIGH ACCURACY*

By J. S. PRESTON, M.A., A.M.I.E.E., F.Inst.P. and
F. W. CUCKOW, B.Sc.
National Physical Laboratory

NOTE ADDED 12 DECEMBER 1936

Our attention has been drawn to a note on the valve bridge published by Dr J. C. M. Brentano in *Nature*, Lond., **108**, 532 (1921). We were unaware of this note at the time of writing the above paper, but of course acknowledge its priority to other publications there referred to.

However, we had in mind strictly only the practical aspects of operating such a valve bridge, and there is nothing in Dr Brentano's note, or in his further papers on the subject,† which affects in any degree the conclusions at which we arrived in regard to the actual circuit which we used. Although Dr Brentano stresses the importance of anode-circuit compensation, we agree with Wynn-Williams that filament-circuit compensation is much more important in securing stability of operation. In fact the theoretical treatment would indicate that with high enough values of anode resistance, and correspondingly high h.t. voltage, the bridge is almost unaffected by small variations in h.t. voltage, even when the anode circuits are not compensated. This was certainly the case with the set-up which we used.

We also regard the setting of the grid of the amplifying valve at the zero grid-current point as of great importance in securing the optimum stability with the type of valves which we used. Since the grid of the other valve can be set at any desired potential to balance the bridge, compensation can still be carried out as desired.

We would finally point out that the references given in our paper are not intended to be in any sense exhaustive, and that we make a definite disclaimer as to novelty.

---

* *Proc. phys. Soc.* **48**, 869 (1936).

† *Phil. Mag.* **7**, 685 (1929); *Z. Phys.* **54**, 571 (1929); *Z. Phys.* **70**, 74 (1931).

THE RESPONSE OF THE MAMMALIAN COCHLEA TO PHASE-REVERSAL IN A CONTINUOUS MUSICAL TONE

By C. S. HALLPIKE, H. HARTRIDGE and
A. F. RAWDON-SMITH,

The Ferens Institute of Otology, Middlesex Hospital, London, W. 1.,
and Physiology Department, St Bartholomew's Hospital
Medical College, London, E.C. 1.

Report given with demonstration, 27 November 1936

ABSTRACT. By means of a photocell, light source and interrupter, it has been found possible to produce phase-reversals in a musical tone of approximately sinusoidal waveform. When this tone is caused to fall on the human ear, phase-change beats are heard as has previously been described. When this tone is caused to fall on the ear of a decerebrate cat, and its auditory-tract responses are recorded by means of a cathode-ray oscillograph, a temporary and marked decrease in amplitude of the response occurs, corresponding to each phase-change. This decrease in amplitude corresponds to the phase-change beat which is heard by the human subject and is postulated on the resonance theory of hearing. When the cochlear (Wever-Bray) response of the cat is similarly recorded, the response exhibits no such fall in amplitude in correspondence with the phase-change, which is moreover reproduced with marked fidelity. The cochlear response must therefore originate in some non-resonant structure.

In previous papers by Hartridge(1) evidence has been advanced that a beat, which has been described as a temporary fall in response, is heard by the human subject on changing the phase of a musical tone by π (180°). This evidence has been contested by Békésy(2), who used a method employing current-reversal in telephones; it has also been disputed by L. Hartshorn(3), who used a similar method. It appeared to us to be desirable to obtain, if possible, objective evidence of the existence of such a temporary arrest in the response. The method we have used was to cause a musical tone, which was changed in phase when required, to fall on the cochlea of a decerebrate cat. The changes in potential which are thus set up in the cochlea were amplified and recorded photographically by means of a moving film camera and a cathode-ray tube. The changes in potential which were set up in the auditory tract, as a result of the passage of nervous impulses, were also similarly recorded during the application of the tone.

The method employed by Hartridge for generating the tone which was changed in phase was to blow a jet of air against uniformly placed slots cut in a rotating brass siren disc; the change of phase was produced by reducing the spacing at one point in the disc to half the distance used elsewhere. This method has the advantage of simplicity, but also the disadvantage of generating marked high-frequency components. These components are in part due to the rush of air from the jet, and in

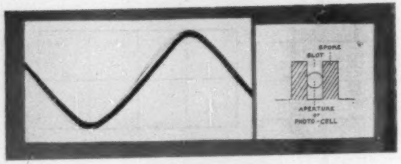


Figure 1.

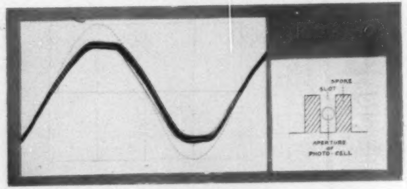


Figure 2.

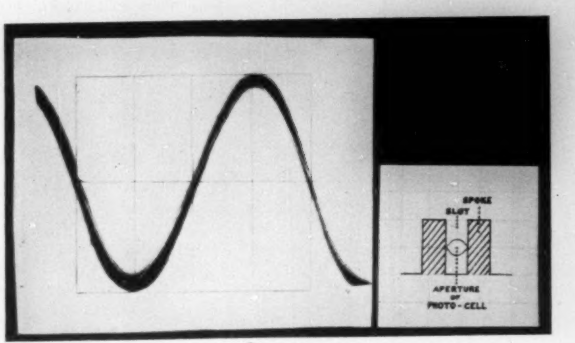


Figure 3.

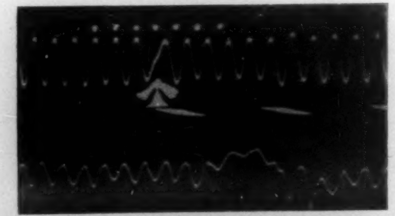


Figure 5. Microphone and auditory-tract response at 1024 c./sec.

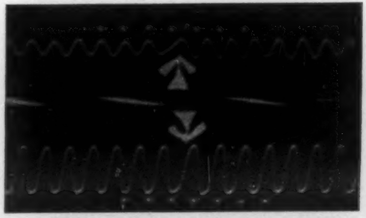


Figure 6. Microphone and cochlear response at 1024 c./sec.

To face page 191

part to the eddies set up by the passage of air through the siren. A method of generating a tone free from such components was therefore looked for.

It was found that if a beam of light was passed through the slots of the siren, and if this interrupted beam was caused to fall on a photocell, the voltages thus set up, when amplified and fed into a moving-coil loud-speaker, resulted in a tone free from these high-pitched components. A record of this tone taken from a piezo-crystal microphone showed, moreover, that it was of roughly sinusoidal form. By attention to certain details, it was found that the wave-form could be further improved. First, a metal-filament lamp and planoconvex lens were adjusted to produce an approximately parallel beam of light of suitable diameter. A circular disc of cardboard, 12 in. in diameter, was marked with a circle 11½ in. in diameter, and this was divided into 80 equal parts. Radial lines were then drawn through the dividing marks. Using these radial lines as contours, alternate slots and spokes of equal width were now marked out, and the slots were cut out with scissors. This disc was mounted between metal plates on a rotating shaft. The parallel beam was arranged to pass through the slots in the disc on to the photocell. When a restricting aperture equal in diameter to the width of the slot was placed in front of the photocell, the voltage changes were found by experiment to follow the black curve shown in figure 1. Calculation of the area of illumination of the photocell at successive instants gave the curve (fine continuous line) which will be seen superimposed on the black one. The two curves coincide as well as might be expected, and both will be seen to have sharper crests and troughs than a sine wave (dotted line). The observed and calculated curves obtained by using a circular restricting aperture of three-quarters the width of the slots (and spokes), which are shown in figure 2, will be seen to have flat crests and troughs. The observed and calculated curves obtained by using a restricting aperture which was lemon shaped, which are shown in figure 3, will be seen to correspond closely to that of a sine wave.

The output of the photocell was fed into an amplifier and loud-speaker. The change of phase was effected at one point of the disc by replacing the usual spoke-slot alternation by two successive slots, or by two successive spokes. Tones of different pitch were obtained by changing the rate of rotation of the disc by means of a continuously variable gear. With this apparatus, which is shown diagrammatically in figure 4, observations were made on (1) the effect of the phase-reversal of a pure tone on the human ear, (2) the effect of the phase-reversal on the electric responses from the cochlea of a decerebrate cat, and (3) the effect of the phase-reversal on the electric responses from the auditory tract of a decerebrate cat. Each of these observations will now be briefly described.

(1) *The response of the human ear to a phase-reversal.* Hartridge has described the beat produced by a change of phase of an impure musical tone as a sharp fall in intensity followed by a rapid return of the intensity to the normal value. He stated that the temporary depression of intensity is often accompanied by a noise of short duration which produces the effect of a tap or thud. The depression of the intensity of the tone and its partial replacement by a noise has been described as "the phase-change beat". We have repeated Hartridge's observations with pure tones of

different frequencies and intensities. The sound stimulus has been analysed by means of a piezo-microphone, amplifier, and cathode-ray oscillograph. Photographic records show that the phase-reversal was effected without significant disturbance of frequency or amplitude. Nevertheless, the audibility of the phase-change beat was undiminished. It was therefore concluded that the beat must be attributed to the phase-reversal and to this alone.

(2) *The response of the auditory-tract potentials to a phase-reversal.* The musical tone generated by the phase-change siren was led from the loud-speaker by thick-walled rubber tubing simultaneously to a crystal microphone and to the external ear of a decerebrate cat. The output of the microphone, after suitable valve amplification, was recorded by means of a cathode-ray tube and moving-film camera. Electrodes were placed in the auditory tract of the cat and the variations of potential recorded from these were similarly amplified and photographed. The two records were taken side by side on the same film together with a time-marker. A record obtained in this way is shown in figure 5. It will be noticed that the microphone faithfully records the phase-change, but that the auditory-tract potentials do not do so. They show, on the contrary, a temporary fall in amplitude of the response, followed by a rapid recovery to the original value. This finding must be considered to indicate that the vibrating elements of the internal ear in which these auditory-tract potentials originate undergo a period of arrested motion in response to a phase-change. This is in agreement with the hypothesis of Helmholtz according to which these elements are regarded as resonant structures. In addition, the view of Hartridge is confirmed according to which a silent period of this kind has been previously postulated as constituting the physiological basis of the phase-change beat.

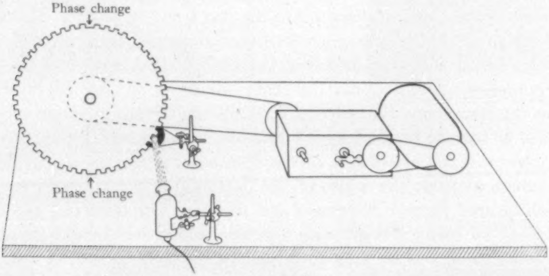


Figure 4.

(3) *The response of the cochlear phenomenon to a phase-reversal.* The cochlear phenomenon was recorded from a platinum electrode placed upon the round window (with an indifferent electrode in the neck muscles) of a decerebrate cat. The output of the cochlea and of the microphone were simultaneously recorded as described above. A comparison of the records, figure 6, shows the marked similarity between

the two, for both follow the phase-change with considerable fidelity. As the microphone consists of carefully chosen parts, light in weight and free from periodic properties, this suggests that the cochlear response originates from structures in the ear with similar properties. If figures 5 and 6 be compared it will be noticed that the tract and cochlear responses are quite unlike one another. Since the tract response originates in nerve endings in the hair cells attached to the basilar membrane, the hypothesis of Davis(4) that the cochlear response originates in these hair cells is rendered unlikely.

REFERENCES

(1) HARTRIDGE, H. *Brit. J. Psychol.* **12**, 142 (1921); *Proc. phys. Soc.* **48**, 145 (1936).
(2) BÉKÉSY. *Phys. Z.* **29**, 793 (1928).
(3) HARTSHORN, L. (Personal communication).
(4) DAVIS, H., DERBYSHIRE, A. J., LURIE, M. H. and SAUL, L. J. *Amer. J. Physiol.* **107**, 311 (1934).

DEMONSTRATIONS

THE PORTABLE PHOTOELECTRIC DAYLIGHT-FACTOR METER.* *Demonstration given by* GEO. P. BARNARD, B.Sc., A.Inst.P. 11 December 1936.

ADEQUATE natural lighting is, for most buildings, even more important than a satisfactory system of artificial lighting. Insufficient natural lighting has often been an important factor in recent years in the condemnation of dwellings and schools.

In consequence of the great variability of daylight, the structural efficiency of a building in admitting daylight to a given point is expressed by the *daylight factor*, i.e. the ratio (usually given as a percentage) of the illumination at the internal point to the simultaneous illumination at an outside point exposed to a complete hemisphere of sky. The difficulties encountered by public health officers in determining the daylight factor have led to a demand for a simple, portable meter.

The portable photoelectric meter shown is designed to measure daylight factors of from 0·03 to 2·50 per cent. Two rectifier photoelectric cells are used. One is placed on the window sill to receive light from a definite fraction of the whole sky-vault; the other at the point in the room where the daylight factor is required. A Morse key facilitates rapid change-over of a microammeter from one cell to the other. Depression of the key connects the microammeter directly to the terminals of the inside-illumination cell. Release of the key connects the microammeter to the outside-illumination cell, in parallel with a variable resistance acting as a shunt to the microammeter. This resistance is adjusted until depression or release of the key produces no change in the microammeter readings; the value of the resistance at balance gives the daylight factor directly on the curve provided.

Corrections for non-linearity in {current, illumination} relationships and for errors arising from the difference in colour between the illuminations to be measured by the two cells are incorporated in the final curve relating shunting resistance with daylight factor. The inside-illumination cell is compensated for loss of sensitivity with increasing obliquity of the incident light.

In operation it is necessary only to connect the cells in circuit, the remaining instruments being permanently wired and mounted in a portable case which has compartments to hold the photoelectric cells and their holders and the necessary flexible leads.

THE AUDIBLE EFFECT OF A SUDDEN CHANGE OF PHASE IN THE CURRENT SUPPLIED TO A TELEPHONE RECEIVER. *Demonstration given by* L. HARTSHORN, D.Sc. 27 November 1936.

THESE experiments arose directly from a demonstration given to the Physical Society last November by Prof. H. Hartridge, F.R.S., who showed that when a siren is arranged to generate a train of sound waves in which there is a reversal of phase at regular intervals without change of amplitude, a listener observes a kind of beat. This is interpreted as a momentary diminution, consequent upon the change of phase, of the amplitude of those vibrating parts of the ear which are in resonance. The experiment was put forward as evidence of the existence of resonance effects in the ear.

---

* *J. Sci. Instrum.* **13**, 392 (Dec. 1936).

The audible effect of the reversal of phase was most marked and I was very surprised when Dr Beatty stated, in the course of the discussion which followed, that Békésy had performed a similar experiment with electrical technique and had obtained a contrary result. It occurred to me that it ought to be possible to make the electrical experiment at least as convincing as the pneumatic one, and that it would be worth while to compare the two. The experiments now to be shown are the outcome of this idea.

A convenient circuit for the production of a variation of phase with no change of amplitude is that shown in figure 1*a*. The corresponding vector diagram is shown in figure 1*b*. The two resistors *R* are equal and non-inductive, *S* is also a non-inductive resistor and *R* a variable condenser. If V_1 is a constant applied voltage, then the voltage V_2 always has the value $V_1/2$, but varies in phase as *C* is varied. This is immediately seen from the vector diagram. As *C* is varied the point *P* travels round the semicircle, having the vector V_1 as diameter, and V_2 is represented by the radius of this semicircle which passes through *P*. Thus any desired change of phase of the voltage V_2 can be produced by a change of capacitance *C*. The actual change produced by any given condenser will of course depend on the values of *C*, *S*, and the frequency, and to produce a change of phase of 180° one must vary *C* instantaneously from zero to infinity. In practice, zero capacitance is an open circuit and infinite capacitance a short circuit. It follows that a complete reversal of phase can be produced by replacing *C* by a tapping key, and simply opening or closing this key. This is the arrangement shown, and the reversal is demonstrated by recording the voltage wave with a cathode-ray oscillograph. On pressing or releasing the key, the voltage wave is seen to be inverted without appreciable change of amplitude. The source of voltage V_1 is a beat-tone oscillator the frequency of which is easily varied from 50 cycles per sec. or less to 10,000 cycles per sec. The arrangement is therefore very convenient for work of this kind, especially as the amount of change of phase is easily controlled.

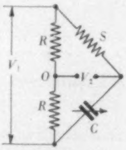


Figure 1*a*.

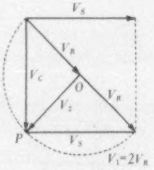


Figure 1 *b*.

The values of *R* and *S* must of course be chosen to suit the work in hand. Those actually used for the demonstration are shown in figure 2. The voltage V_2 is amplified by a resistance-capacity-coupled amplifier. Inductances and transformers were avoided since their behaviour might confuse the issue. The two resistances *R* each have the value of 100 ohms. The resistance *S* is one hundred times larger, and a second resistance of 10,000 ohms is placed between the oscillator and the bridge network. In this way the voltage V_1 applied to the bridge network is, to an accuracy of the order of 1 per cent, independent of changes from zero to infinity of the impedance of the variable arm of the network. The input impedance of the amplifier is very high compared with any of the resistances of the network and may therefore be regarded as infinite. The conditions assumed in figure 1 are therefore satisfied.

The output of the amplifier is connected to a cathode-ray oscillograph, in parallel with which we may, by the operation of a selector switch, connect a vibration galvanometer, telephone earpiece, or loud-speaker. Thus the reversal of phase is seen in the cathode-ray oscillograph at the instant when its effect on the ear via the telephone or loud-speaker is heard.

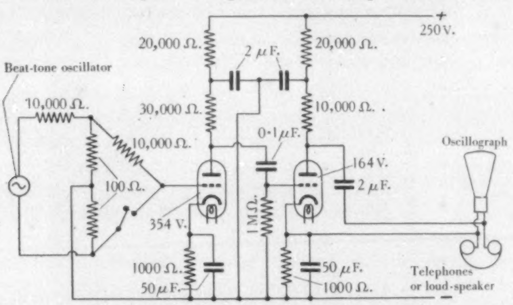


Figure 2. Phase-changing circuit and amplifier.

The experiment was first shown with an ordinary loud-speaker of the moving-coil type. On operating the key and reversing the phase a pronounced click was heard. The experiment was then repeated with a piezo-electric loud-speaker. Again a click was heard, but of a somewhat different character.

The experiments were repeated at various frequencies between 200 and 10,000 cycles per sec. In all cases there is a very pronounced discontinuity in the sound heard, but it is not easy to describe with any precision the nature of this discontinuity. Some observers describe a momentary cessation of the tone heard; others an increase in the loudness of the tone immediately following the change of phase. Others merely a click or a ping such as might be expected to result from a very short impulse applied to the diaphragm of the instrument under observation. Whatever the ultimate explanation may be, there seems to be no reason to suppose that the effect heard is essentially different from that demonstrated by Prof. Hartridge with his original siren and with the photoelectric siren which he has just shown. There can therefore be no question of a contradiction between the results of the experiments made with electrical and with purely acoustic technique. It is only in the interpretation of the result that differences of opinion may arise.

In this connexion the following experiment is interesting. The loud-speaker is replaced by a vibration galvanometer which is tuned to resonance with the applied voltage. On reversing the phase the amplitude of vibration diminishes nearly to zero and then rises to its original value, as is to be expected. If, however, the galvanometer is tuned to be just off resonance, either by increasing or diminishing the frequency, the reversal of phase is seen to cause a temporary *increase* in its amplitude of vibration.

Thus if we suppose the ear to possess a series of resonators graded in frequency, although the amplitude of those which are in tune will be diminished by a reversal of phase, the amplitude of the resonators with frequencies either a little greater or a little less than that of resonance will be increased. It is not therefore surprising that few observers describe a silent period, and it is possible that the temporary increase of intensity sometimes heard is due to this increased response of out-of-tune resonators. Such resonance effects may be due to the loud-speakers as well as to the ear.

Although I do not propose to attempt to discuss the theories of hearing there is one other point which I would emphasize. It is obviously physically impossible to change suddenly the phase of a continuous train of waves without at the same time introducing a transient vibration into the system. Its character may vary according as the change is made at the instant of zero displacement, of zero velocity, or some intermediate point, but it must always be there, and it is possible that when the experiment is made it is this transient vibration which attracts the attention of many observers. It is true that the visual examination of the cathode-ray oscillograph fails to detect the transient with the apparatus now being demonstrated, while the audible effect is most marked. Nevertheless the effect heard frequently suggests a sharp blow on the vibrating system and it is possible that the audible effect of the transient may be more prominent than that of the change of phase.

It is clear from the recent advances made by Prof. Hartridge and his collaborators that much work remains to be done before the theory of hearing will be completely elucidated. The only claim made for the experiments which I have shown is that they might provide a convenient technique for the carrying out of some of the work.

REVIEWS OF BOOKS

Recollections and Reflections, by Sir J. J. THOMSON, O.M., F.R.S. Pp. viii+451. (London: G. Bell and Sons, Ltd.) 18*s*. net.

Here are riches for many types of reader, all of whom will feel and be impressed by the spirit of happiness which runs as *Leit-motiv* through this record of a fruitful life spent in ideal surroundings with hosts of pleasant friendships. "I came up to Trinity College in October 1876 and have kept every term since then, and been in residence for some part of each long vacation." Such long and intimate association with Cambridge enables Sir J. J. Thomson to survey the many changes which he has seen in University life and in the progressive development of teaching and research, and to comment understandingly upon such different topics as the status of women students and the changing popularity of various games. He can and does give us pen pictures of Cambridge men whose names are as household words alike to scientific worker and to general reader. Here also we find appreciative sketches of his predecessors in the Cavendish Chair of Experimental Physics, of Clerk Maxwell (1871–9) with an amusing reminiscence of his first lecture on heat (Sir Joseph's laconic comment on Maxwell's report that the laboratory "now contains all the instruments required by the present state of science" must not fail to bear fruit), and of Lord Rayleigh (1879–84) whom Thomson succeeded at the age of 28. What could be more romantic than his account of Prof. H. L. Callendar, "an intellectual Admirable Crichton whose career at the Laboratory was in some respects the most interesting in all my experience"! In the last hundred pages Sir J. J. Thomson gives some account of the development of that branch of scientific work conveniently classified under the heading of the discharge of electricity through gases.

There is a wealth of good stories, but of set purpose I will quote none, for I am certain that, owing to the interval which must necessarily elapse between the writing of this notice and its date of publication, most if not all of the readers of the *Proceedings* will have read and reread these charming reminiscences of our great physicist.

If I can generalize from my own experience, extending over a period of one week only, I know that the place on our shelves reserved for this fascinating autobiography will rarely house its rightful occupant because we shall have been forced to yield to the loan-requests of our friends in every walk of life.

Gilbert of Colchester has been called the father of the sciences of electricity and magnetism. These sciences have been developed so greatly during the last forty years, largely owing to the work of Thomson at Cambridge, that it would certainly seem more appropriate to put Gilbert one stage further back in the table of consanguinities and to recognize Sir J. J. Thomson as the father of these sciences as we know them to-day. Perhaps we younger members of the world-wide but united family of physicists have unconsciously recognized his paternal standing, for alone amongst scientists with international reputations he stands as a man known by initials which fall trippingly from the tongue, a hero of science respected, admired and held in deep affection, simply as J. J.

J. H. B.

An Introduction to Nuclear Physics, by N. FEATHER, Ph.D. Pp. x+213. (Cambridge University Press, 1936.) 10*s*. 6*d*. net.

To summarize the present position of a subject which is expanding at such a prodigious rate as nuclear physics is no light task, as may be appreciated from the fact that the present volume contains references to some 1200 papers, most of them published within the last three years and not a few within the last twelve months. Dr Feather may be congratulated on his courage in undertaking this much-needed work, and on the ability with which he has carried it out.

In spite of the modesty of its title, the book provides a very adequate critical summary of the present state of knowledge of the nucleus as a structure, and of the transformations produced in it by fast-moving particles and high-energy quanta. No research of importance appears to have escaped notice, and all the important results are exhibited in tabular form. The book is entirely non-mathematical in character, and while the various hypotheses and concepts which are being used at the moment to co-ordinate and interpret the experimental data are adequately described, the author has very wisely eschewed all proofs, and is content simply to quote such formulae as are required for his exposition.

The subject-matter is well arranged and admirably documented; the addition of a subject index would have made easier the task of those who wish to consult the book on special points. The book does not altogether make easy reading, and we can imagine that the average honours student might find it rather beyond his unaided ability. To post-graduate students, and to physicists generally who wish to be *au fait* with this newest branch of their subject, the book can be unreservedly recommended.

J. A. C.

The Electron. (1) *Einführung in die Elektronik*, by O. KLEMPERER. Pp. xi + 303. (Berlin: J. Springer, 1933.) RM. 18.60. (2) *Elektronentheorie der Metallen*, by H. FRÖHLICH. Pp. vii + 386. (Berlin: J. Springer, 1936.) RM. 27.

These two books are in some ways complementary, that by Klemperer having an experimental outlook and that by Fröhlich a mathematical one, although the distinction is by no means absolute. Each contains some description of experimental work and some account of the theoretical basis of the treatment.

Klemperer deals first with electronic beams and the means of imparting high velocities to them, whereas the first step in the book by Fröhlich is a short treatment of the Schrödinger wave equation. In subject matter, too, they are in some respects complementary, though there is more overlapping than the titles would indicate. Klemperer includes in his range the properties of electrons in metals and has a chapter on electron gas in which velocity-distribution, Fermi statistics, and degeneration are dealt with, even though rather shortly. Moreover, about one third of his book is concerned with electron-emission, and here he is necessarily dealing largely with the indirect evidence as to the properties and behaviour of electrons in metals, as evidenced by the phenomena of the photoelectric, the Richardson and the cold-emission effects. Conversely, Fröhlich, although mainly concerned with electrons in metals, does on occasion refer to the properties of free electrons, generally as illustrations of the mathematical theory. The main part of his book is concerned with quite recent work. His starting point after the classical paper by Pauli (1927) is that by Sommerfeld (1928), in which the Drude-Thomson theory was recast in terms of Fermi statistics. (In passing, one must regret that the charmingly written paper by Sommerfeld, in *Die Naturwissenschaften* for 1927, in which he gave a preliminary account of these results, seems doomed to be forgotten, hidden by the more complete paper of the following year.)

After the introductory and historical matter, Fröhlich takes in turn the problems of conductivity, semiconductors, the chemical bond, and magnetism, following which there is a final chapter discussing the metals as a whole in their relationship to the periodic table.

The treatment is up-to-date (Mott's work of 1936 is included) and accurate, but the book gives an impression of superficiality which is not really justified; this is caused by the rapid transition from subject to subject, with no halts to explain at length the implications of results, though they are there to be dug out by the reader who is prepared to go slowly enough to do the digging himself.

Klemperer's book, which covers a much wider range, can best be exhibited by giving a somewhat breathless list of some subjects included in it. They are all well expounded, though the outlook of 1937 is not that of 1933 on such matters as the numerical value of the electronic charge, or the formulation of electron optics. Here then is the list: the behaviour of electrons in rectifiers, amplifiers, valves, X-ray tubes, photocells and oscillographs; Fermi statistics; refraction, diffraction and polarization of electrons; electron optics, velocity spectra, the cloud chamber, counters, sensitivity of photographic plates to electrons, space-charge effects, plasma; thermionics and cold emission, the photoelectric effect, selective and otherwise, exit work function, ionization potentials, and the grouping of electrons in the atoms of the elements; the Compton effect, secondary electrons (a very thorough and informative chapter of 21 pages); ionization by ions, including positrons and α particles; radioactivity, with no mention of the neutrino; 100 pages on interaction between atoms and free electrons, in which are introduced the wave properties of the electron, and a very interesting discussion of the individual processes by which electrons lose energy in passing through matter; single and multiple scattering. From this it will be seen that the book is very thorough. It is notable for the many numerical tables, and is in every way a useful book to have about the laboratory—even more perhaps than in the study.

J. H. A.

Television Optics, by L. M. Myers. Pp. x + 338, 214 figures. (London: Sir Isaac Pitman and Sons, Ltd., 1936.) 30*s*. net.

The appearance of a book on television optics is timely. From different standpoints many members of the general public, students of physics, and workers in one or other of the branches of physics utilized, are interested in learning what is being done in this subject and if possible in understanding the principles employed. To make the widest appeal, and perhaps it may be added to be of the widest scientific service, such a book should be as free as possible from mathematical detail and from mere technicalities. Words used with meanings which are not found in a good dictionary of normal size should be carefully explained. The traveller should not be allowed to wander from the road he has to traverse; his desire, it may be safely assumed, is to understand the special development with which the book deals, not to receive on the way a general education in associated branches of science. Liberal illustration of the text is a valuable feature. In the last respect this book is satisfactory, but in many others it appears to the reviewer to leave much to be desired. It will certainly not be understood, as it might have been, by readers without a special training in science. Readers in general would have been better served by the exclusion of excrescences which are not needed for understanding the ultimate aim of the book. It should not have been difficult to delete a substantial portion of the text without detriment to the rest, and so to lower the price appreciably. In addition the author must be blamed for not taking the trouble to find what are the accepted meanings of some of the technical terms he uses. In the older branches of physics, at any rate, there is no doubt what is to be understood by the more important words, but it would be possible from this volume to illustrate the application of well-known terms to well-known phenomena with which they have nothing to do. For these reasons the reviewer strongly dissents from the advertisement on the jacket, which says "This is an essential book for students and others who need a thorough grasp of the theory underlying this rapidly developing branch of applied science. It is noteworthy for the simplicity of its mathematical analysis and for the elimination of unnecessary practical matter already available from other sources...."

In spite of these defects the book will serve the purpose of indicating what has so far been achieved, on what lines work is being pursued, what are the difficulties encountered, and why development is proceeding in this direction rather than that. This is a proper function for a book dealing with the present state of a subject still in its infancy. For the time being it will rightly be given a place in scientific libraries, but it should be possible before long to replace it by a book free from defects such as have been mentioned.

T. S.

Vibration and Sound, by PHILIP M. MORSE. Pp. xv+351. (London: McGraw-Hill Publishing Co., Ltd., 1936.) 24*s*.

Prof. Morse is at some pains in the preface of his book to find an excuse for adding yet another to the list of books dealing with the subject of vibrations and sound. He need not be apprehensive, however, for the rapid growth in the importance of the subject would be sufficient justification, if such were necessary, for the book he has now published in the International Series in Physics. This book is different from most of the others; both the style and the manner of treatment are different. It is mainly mathematical, but the physics is not forgotten, in fact, as is stated in the preface, one of the principal aims of the book has been to give to the student examples of the *method* of theoretical physics, "the way a theoretical physicist attacks a problem and how he finds its solution".

The subject matter covered by the book is not comprehensive, but what is done has been done thoroughly—even the section headed "A Little Mathematics" in the introductory chapter is most effective in its appeal and proves of considerable value in the later stages of the book. After the introduction, successive chapters deal with the simple oscillator, the flexible string, the vibration of bars, membranes and plates, plane waves of sound, radiation and scattering of sound, and standing waves. A useful bibliography and tables of functions are given in conclusion. The three-dimensional illustrations of the vibrations of membranes and plates, with the series of high-speed motion pictures of waves on a rubber membrane, are valuable aids to a mental grasp of this difficult subject.

The last chapter in the book serves as an illustration of the original manner of treatment. In this chapter the mathematical methods developed for the study of the radiation of light from an atom are applied to the theory of the acoustic properties of rooms. The reviewer notes with pleasure that the book is dedicated to Prof. Dayton C. Miller.

Prof. Morse's book can be thoroughly recommended both as a textbook and as a work of reference.

A. B. W.

Properties of Matter, by F. C. CHAMPION and N. DAVY (*The Student's Physics*, Vol. III). Pp. xiv+296. (London and Glasgow: Blackie and Son.) 15*s*. net.

Unless he sets out to write on the lines of a *Handbuch* (i.e. a book apparently so-called because it cannot be moved by hand, without the aid of one of the six mechanical powers), the author of a text on properties of matter is likely to find more difficulty in selecting than in handling his topics. The authors of this text have dealt pretty thoroughly with a number of selected (and on the whole well selected) topics, and jettisoned others more or less completely. From the standpoint of the reader for whom this series is primarily intended—the student reading for honours in physics—this is undoubtedly the best compromise, and it would be unfair to dwell too heavily upon the resulting omissions.

The absence of any reference, in an otherwise rather comprehensive chapter on capillarity, to the dependence of vapour pressure upon curvature of surface, may be excused on the ground that this particular problem is treated in Roberts's companion volume on *Heat and Thermodynamics*. The sketchiness of the chapter on kinetic theory (in which, for instance, transport problems are handled by the simplified six-stream method of Joule) may be partially justified on the same grounds, though in view of recent work on the separation of isotopes we should have liked to see a little more about diffusion in gases.

The chief omissions are however of outmoded methods of measurement—of, for example, determinations of the gravitational constant of which the interest, if any, is purely historical. The space so saved is more profitably devoted to more modern developments. There is a chapter on seismic waves and another on surface films. The chapter on osmotic pressure contains a brief account of the Milner-Debye-Hückel theory of strong electrolytes (nothing about Onsager's additions to the theory), and that on viscosity a brief note on Andrade's theory. The most unusual feature of the book is a summary of methods for the determination of Planck's constant—which, when one thinks of it, is perfectly in place in such a text.

The book is well and clearly written and illustrated; it should have a wide circulation among university students of physics. H. R. R.

Biological Time, by LECOMTE DU NOÜY. Pp. 177. (London: Methuen and Co., Ltd., 1936.) 7*s*. 6*d*. net.

From a study of the rate of cicatrization of wounds, begun during the war, the author has deduced certain general equations which enable a fairly accurate prediction of the date of the completion of healing to be made. Experiments on the behaviour of tissue cultures *in vitro* also have been examined, and from these studies the author produces the idea of a physiological time which bears a certain relation to ordinary time. The conception arrived at is of a "granular, variable, individual time, differing from the continuous, integral, universal time". This physiological time is said to flow more rapidly with increasing age, the rate increasing in proportion to the age of the individual. The book is divided into three parts: the biological problem, and methods; cicatrization of wounds, and tissue-culture; time. The first part serves as a general introduction and the second gives an account of the research methods used and of the experiments on which the author's deductions are based. These two parts are a mixed grill, written partly in a simple and partly in an elaborate style and including both elementary and advanced knowledge, and they touch on many branches of mathematics, physics, chemistry and biology. Some of the interesting work carried out at the Rockefeller Institute and l'Institut Pasteur, with both of which the author has been associated, is recorded. The last part, on time, is on the whole philosophic and contains many sweeping generalizations. The book is occasionally thought-provoking, often fascinating and sometimes abstruse.

H. R. L.

An Elementary Survey of Modern Physics, by G. F. HULL. Pp. xxiv + 457. (London and New York: Macmillan and Co.) 20*s*. net.

This book is remarkable for its completeness, in the sense that practically no topic of interest in modern physics has been completely ignored. It necessarily follows, given the size of the book, that where so many topics have been introduced the majority of them have had to be rather summarily dismissed. As a rule, however, enough has been said to entertain and to stimulate the interest of the average reader.

The treatment is livelier than is customary in English textbooks, and there is a more lavish use of exclamation marks. We quote a few headlines:

"After Seventeen Years a Great Discovery Is Made. X-Rays Are Wavelike."
"The Neutron Enters an Equation for the First Time."
"As a Ghost It Enters Photographs."
"Rush Hour in Twentieth Century Physics."

The treatment is well adapted to the general reader, though in one place "the student" is warned that he "must now be prepared for a headache". The book is (except for a gratuitous and unjustifiable attack on "Rayleigh-Jeans") pleasantly and genially written; it should find favour with a wide circle, even if it does not quite completely equip budding scientific journalists, and if it fails occasionally to enable "students of philosophy" to "confound their opponents". (In the reviewer's experience, a knowledge of physics is not an indispensable passport to journalism, and philosophers are not easily persuaded that they have been confounded.)

The book is well and lavishly illustrated with photographs and line diagrams. Reference has already been made to the width of the field under survey; this may also be illustrated by listing the last six of the many appendices, viz. (1) Schroedinger's equation, (2) A brace of the Cavendish Society's post-prandial dirges, (3) A French mnemonic for π to 30 places, (4) A numerical example on the uncertainty principle, (5) A list of physicists who have been awarded Nobel Prizes (Query: is this to remind us that "Tout soldat français porte dans sa giberne le bâton de maréchal de France"?), and (6) A short list of important physical constants.

The treatment is avowedly elementary, and indeed it is clear that mathematical discussions do not fit comfortably into the general scheme. In spite of this, the book, because it is thoroughly up-to-date both in its information and its viewpoint, will be of value to confirmed physicists as well as to the general reader. This is especially true of such sections as those on spontaneous and induced atomic disintegrations, in which the author has been relatively prodigal of his available space.

On the whole a book which deserves, and will probably obtain, considerable success.

H. R. R.

Mercury Arcs, by Prof. F. J. Teago, D.Sc., M.I.E.E. and J. F. Gill, M.Sc. Pp. vii + 104. (London: Methuen and Co., Ltd., 1936). 3*s*. net.

This monograph deals with the mercury arc as a means of converting alternating current to direct current on a large scale. It will be of considerable interest, particularly to electrical engineers, covering concisely as it does the action of this apparatus from the electrical, mechanical and operational standpoints. Since Cooper-Hewitt in 1903 first utilized the rectifying property of mercury vapour, the development of the mercury rectifier has been very rapid, mainly owing to the advent of grid control. Units of capacity up to 2500 kW. at 3300 V. are now in operation, water-cooled containers being employed for sizes above 250 kW.

After an interesting introductory chapter, which includes an exposition of the physical phenomena involved, the authors deal with constructional details, grid control of voltage, grid control of current, {current, voltage} characteristics, and the curious phenomenon of inversion leading to the practicability of interconnexion between alternating current mains of different frequencies through the intermediary of mercury rectifiers. Chapters on wave-form analysis, transformer ratings, and power factor complete a treatment in which mathematical analysis is a prominent feature.

D. O.